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SPECIAL AMMENSAL MODEL WITH MONAD COEFFICIENT - A LOGICAL STUDY

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Abstract: The paper is intended to discuss about the influence of One of the Monad parameters(c) on the stability nature of Ecological Ammensalism based on the threshold results. This model is considered by a couple of first order non linear differential equations with influencing Monad parameters. The behavior of this model is observed with Phase plane analysis.

Keywords and Phrases: Ammensal species, Enemy Species, Equilibrium points, Stability and threshold diagrams.

2010 Mathematics Subject Classification: 92D25, 92D40.

1. Introduction and Preliminaries

Many real ecological models are examined with the concepts of Mathematical modeling to gain suitable existence solution in many complex cases. Initially ecological models were discussed by Lotka [11] and Volterra [14]. Many mathematicians Meyer [12], Cushing [6], Gause [8], Paul Colinvaux [13], Haberman [9], Freedman [7], Kapur [10] etc investigated various models and their stability. Later

K. V. L. N. Acharyulu et.al [1-5] concentrated on stability analysis in multiple cases of Ammensalism. The behavior of Ecological Ammensalism with monad coefficient is established with Phase plane analysis.

Notations Adopted.
x (t) : The population rate of Ammensal Species(S1) at time t y (t) : The population rate of Enemy Species(S2) at time t u : The natural growth rate of Ammensal Species(S1) v : The natural growth rate of Enemy Species(S2) a :The inhibition coefficient of Ammensal due to Enemy Species M(y) : Monad Coefficient which is defined with two parameters i.e $M(y) = \frac{by}{c+y}$ The state variables x and y as well as the model parameters u, v, a, e are assumed to be non-negative constants.

2. Results

(i). Equation for the growth rate of Ammensal Species (S1):
$\frac{dx}{dt} = ux - ax^2 - x\ M(y)$ where $M(y)$ is Monad coefficient and defined by

$$M(y) = \frac{by}{c+y} \tag{1}$$

(ii). Equation for the growth rate of Enemy species (S2):

$$\frac{dy}{dt} = v\ y - e\ y^2 \tag{2}$$

3. Equilibrium states

The system has the following four equilibrium states from $\frac{dx}{dt} = 0$ $\frac{dy}{dt} = 0$
(i). Fully washed out state:

$$\bar{x} = 0, \bar{y} = 0. \tag{3}$$

(ii). The state in which only the enemy survives and the Ammensal is washed out:

$$\bar{x} = 0, \bar{y} = \frac{v}{e} \tag{4}$$

(iii). The state in which only the Ammensal survives and the enemy is washed out:

$$\bar{y} = 0, \bar{x} = \frac{u}{a} \tag{5}$$

(iv). Co-existent state: (Both Ammensal and enemy exist)

$$\bar{y} = 0, \bar{x} = \frac{1}{a}\Big[u + \frac{(b\ \frac{v}{e})}{c + \frac{v}{e}}\Big]. \tag{6}$$

Now , the nature of this model is discussed with threshold diagrams with the considered values for different parameters. If $c > 0$ which lies between 0 and 1,then the influences on the system are observed.

Case(i): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.05. The Null clines and Trajectories are shown in the Fig.(1) and Fig.(2) respectively. In this case, The Eigen values are -0.18638 and -0.76 with the eigen vectors (1,0) , (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.18638 & -7.9004E-11 \\ 0 & -0.76 \end{bmatrix}$. The interaction between the Ammensal and Enemy Species is shown in Fig.(3).

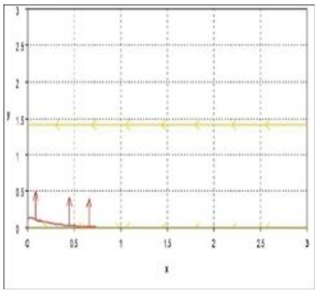

Figure 1

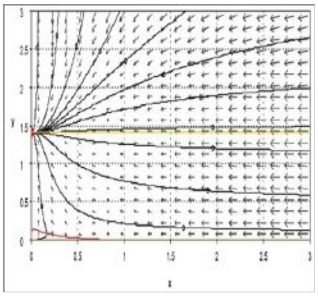

Figure 2

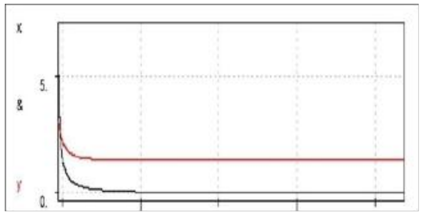

Figure 3

Case(ii): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.15. The Null clines and Trajectories are shown in the Fig.(4) and Fig.(5) respectively. In this case, The Eigen values are -0.12561 and -0.76 with the eigen vectors (1,0), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.12561 & -1.1158E-19 \\ 0 & -0.76 \end{bmatrix}$

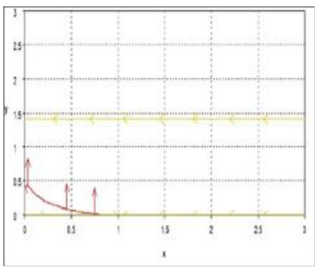

Figure 4

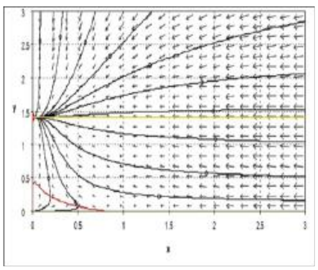

Figure 5

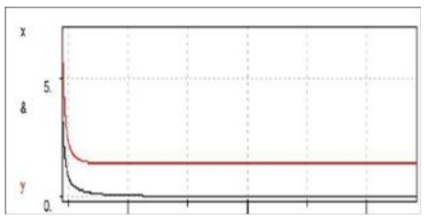

Figure 6

The interaction between the Ammensal and Enemy Species is shown in Fig.(6).

Case(iii). When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.25. The Null clines and Trajectories are shown in the Fig.(4) and Fig.(5) respectively. In this case, The Eigen values are -0.072179 and -0.76 with the eigen vectors (1, 1.32004E-8), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.072179 & -1.1158E-19 \\ 0 & -0.76 \end{bmatrix}$

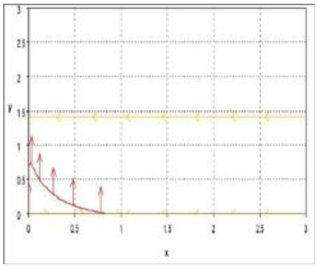

Figure 7

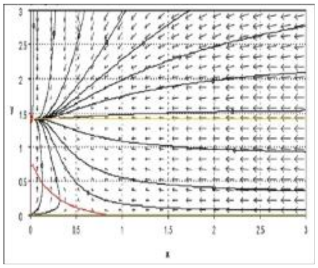

Figure 8

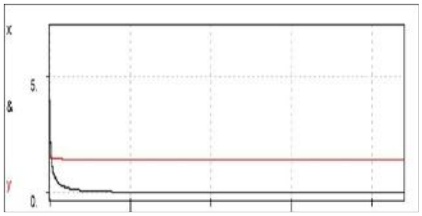

Figure 9

The interaction between the Ammensal and Enemy Species is shown in Fig.(9).

Case(iv): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.35. The Null clines and Trajectories are shown in the Fig.(10) and Fig.(11) respectively. In this case, The Eigen values are -0.024826 and -0.76 with the eigen vectors (1, 0), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.072179 & -4.37685E-12 \\ 0 & -0.76 \end{bmatrix}$

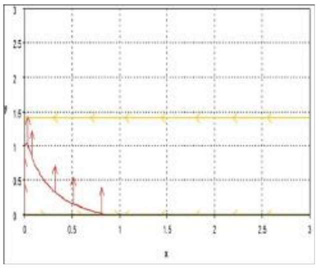

Figure 10

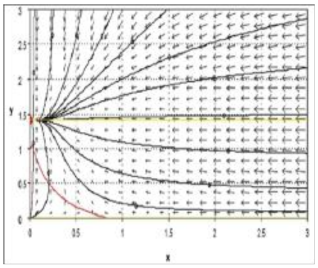

Figure 11

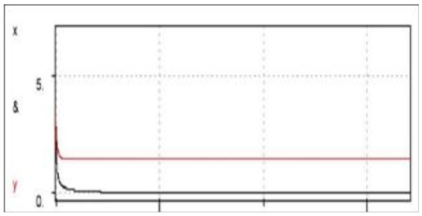

Figure 12

The interaction between the Ammensal and Enemy Species is shown in Fig.(12).

Case(v): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.45. The Null clines and Trajectories are shown in the Fig.(13) and Fig.(14) respectively. In this case, The Eigen values are -0.017428 and -0.76 with the eigen vectors (1, -8.31643E-15), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.017428 & -0.0025031 \\ 0 & -0.76 \end{bmatrix}$ The Equilibrium point occurs at (0.019582, 1.4074).

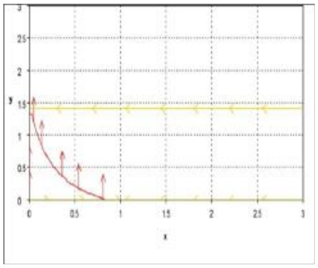

Figure 13

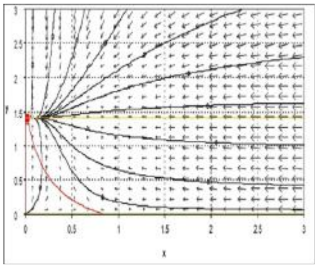

Figure 14

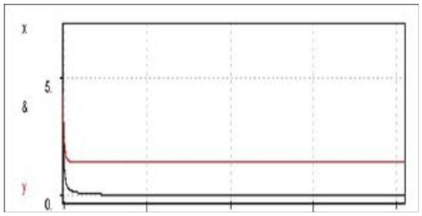

Figure 15

The interaction between the Ammensal and Enemy Species is shown in Fig.(15).
Case(vi): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.55. The Null clines and Trajectories are shown in the Fig.(16) and Fig.(17) respectively. In this case, The Eigen values are -0.055364 and -0.76 with the eigen vectors (1, 1.58582E-15), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.055364 & -0.0087512 \\ 0 & -0.76 \end{bmatrix}$ The Equilibrium point occurs at (0.062207, 1.4074).

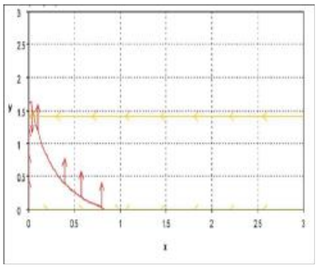

Figure 16

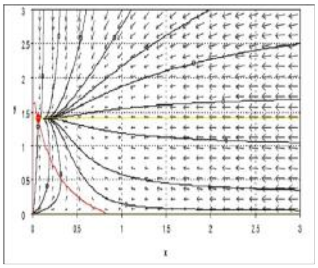

Figure 17

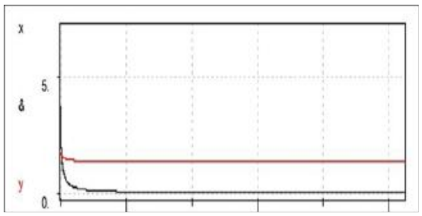

Figure 18

The interaction between the Ammensal and Enemy Species is shown in Fig.(18).
Case(vii): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=.65. The Null clines and Trajectories are shown in the Fig.(19) and Fig.(20) respectively. In this case, The Eigen values are -0.089613 and -0.76 with the eigen vectors (1, -9.15884E-16), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.089613 & -0.015152 \\ 0 & -0.76 \end{bmatrix}$ The Equilibrium point occurs at (0.10069, 1.4074).
The interaction between the Ammensal and Enemy Species is shown in Fig.(21).
Case(viii): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=0.75. The Null clines and Trajectories are shown in the Fig.(19) and Fig.(20) respectively. In this case, The Eigen values are -0.089613 and -0.76 with the eigen vectors (1, -9.15884E-

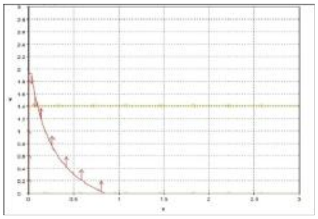

Figure 19

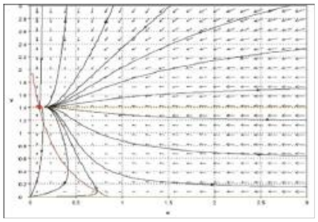

Figure 20

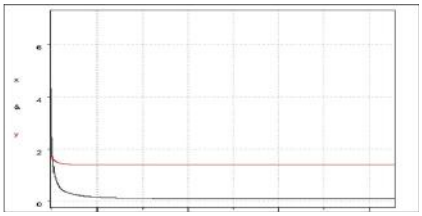

Figure 21

16), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.089613 & -0.015152 \\ 0 & -0.76 \end{bmatrix}$ The Equilibrium point occurs at (0.10069, 1.4074).

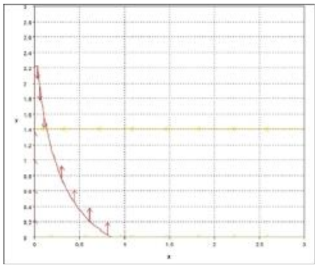

Figure 22

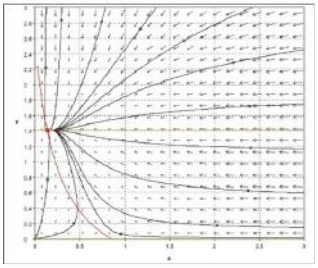

Figure 23

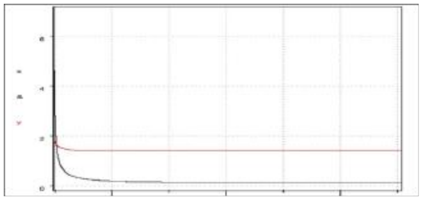

Figure 24

The interaction between the Ammensal and Enemy Species is shown in Fig.(24).
Case(ix): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=0.85. The Null clines and Trajectories are shown in the Fig.(25) and Fig.(26) respectively. In this case, The Eigen values are -0.12069 and -0.76 with the eigen vectors (11.94424E-15), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.12069 & -0.021414 \\ 0 & -0.76 \end{bmatrix}$ The Equilibrium point occurs at (0.1356, 1.4074).

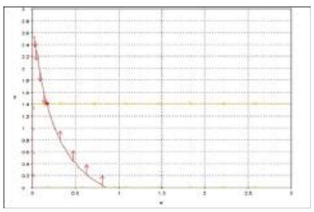

Figure 25

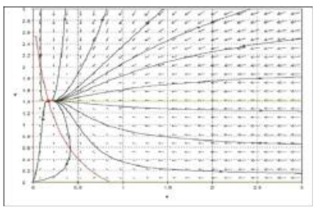

Figure 26

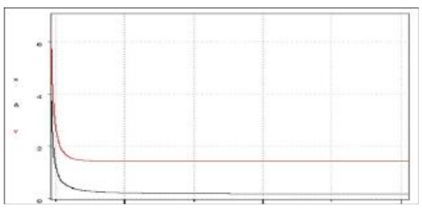

Figure 27

The interaction between the Ammensal and Enemy Species is shown in Fig.(27)

Case(x): When u=0.98, a=0.76, v=0.98, b=0.89, e=0.67 and c=0.95. The Null clines and Trajectories are shown in the Fig.(28) and Fig.(29) respectively. In this case, The Eigen values are -0.17493 and -0.76 with the eigen vectors (1, 8.4296E-16), (0,1) and the Jacobean matrix is $\begin{bmatrix} -0.17493 & -0.032926 \\ 0 & -0.76 \end{bmatrix}$ The Equilibrium point occurs at (0.19655, 1.4074).

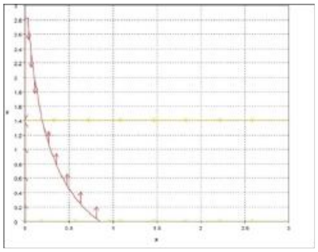

Figure 28

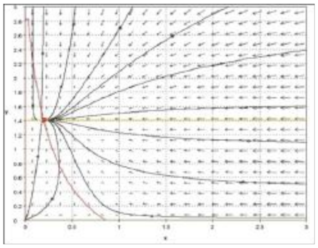

Figure 29

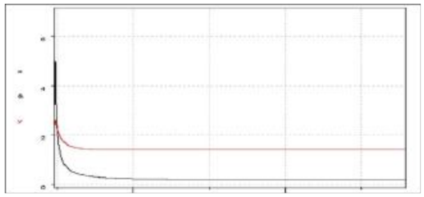

Figure 30

The interaction between the Ammensal and Enemy Species is shown in Fig.(30).

4. Conclusion

The system is stable at coexistence point. The nature can be slightly changed by decreasing the growth rate of Ammensal Species than the growth rate of enemy species to reach a stabilized stable state. Finally it is observed that the Ammensal Species declines gradually than enemy species with the influence of one of the parameters(c) of Monad coefficient in a specified period of time and then becomes a constant rate.

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