*South East Asian J. of Mathematics and Mathematical Sciences*
*Vol. 16, No. 1 (A) (2020), pp. 67-74*

**ISSN (Online): 2582-0850**

**ISSN (Print): 0972-7752**

# FREE AND FORCED CONVECTIVE HEAT TRANSFER THROUGH A NANOFLUID IN TWO DIMENSIONS PAST MOVING VERTICAL PLATE

**B. Sailaja, G. Srinivas*, B. Suresh Babu**, G. Kiran Kumar*****

Department of Mathematics,
Dravidian University, Kuppam, INDIA
E-mail : sailuabbaraju72@gmail.com

*Department of Mathematics,
Guru Nanak Institute of Technology, Hyderabad, INDIA
E-mail : gn.nivas@gmail.com
**Department of Mathematics,
Sreyas Institute of Engineering and Technology, Hyderabad, INDIA
E-mail : bsureshmaths@gmail.com
***Department of Mathematics,
Annamacharya Institute of Technology and Sciences, Hyderabad, INDIA
E-mail : aithkiran@yahoo.com

(**Received:** *Mar. 05, 2020* **Accepted:** *April. 22, 2020* **Published:** *Apr. 30, 2020)*

**Abstract:** The present paper aims to study the convective high temperature transfer of Nanofluids into which we use viscosity proposed with Einstein also with the thermal conductivity proposed by Corcione. Particularly in this paper discussion is about free and forced convective heat transfer in Cu – water Nano- fluid past permeable flat vertical semi-infinite moving plate due to high conductivity and occurrence in Cu-water Nanofluid with natural or forced convections in which we consider magnetic field and also heat source. The effect on various parameters were exhibited in graphs. The profile of every governing parameter is displayed for natural as well as forced convection by considering the $Ar >> 1$ and $Ar << 1$ respectively.
**Keywords and Phrases:** Free and forced, Convective, Heat Transfer, Nano Fluid, Vertical Plate.

**2010 Mathematics Subject Classification:** 80A20, 76R10, 76R10.

## 1. Introduction

Nanofluids consisting of nanometer-sized solid particles and fibers discrete in liquids have recently been demonstrated to have great potential for improving the heat transfer properties of liquids. Some distinctive behavior of nanofluids have been identified, including the possibility of obtaining large increase in thermal conductivity compared with liquids without nanoparticles, strong temperature-dependent effects, and significant increase in vital heat flux. Natural convection heat transfer in nanofluids has begun to receive attention owing to potential applications of nanofluids. Putra et al. [1] studied natural convection of $Al_2O_3$ in water and CuO in water nanofluids. Unlike conduction or forced convection, these natural convection experiments showed a decrease in heat transfer. Khanafer et al. [2] developed a thermal dispersion model to simulate natural convection heat transfer of nanofluids in a two-dimensional enclosure. Eastman et al. [3] reported that the heat transfer coefficient of water containing 0.9 vol % of CuO nanoparticles was improved by 15 % compared with that of water without nano particles. Recently, Xuan and Li [4] measured the convective heat transfer coefficient and friction factor of Cu-water nanofluids in turbulent flow. Pak and Cho [5] found that the convective heat transfer coefficient of water-based nanofluids with 3 vol.% $Al_2O_3$ and $TiO_2$ nanoparticles was 12% smaller than that of pure water when tested under constant average velocity. One possible reason for this reduction in heat transfer could be that the viscosity of the nanofluids prepared in this study was much larger than that of the nanofluids used by Wang et al. [6] as discussed above. In the study of their research it was found uniformly with heated vertical walls in a rectangular cross-section channel in various restricted inlets and outlets these details were obtained from the work done by Neiswanger et al [7] in 1987, Pushpalatha [8] were studied in their research and found free convection flow on casson fluid and identified that the effect is on velocity field. The magneto hydro dynamic Casson fluid in a parallel plates channel of stretching walls was performed by Sugunamma et al [9]. Srikanth et al. [10] were studied the effects of size of nano particles were found in their study of their research the destructive chemical reaction exponentially decreases the velocity. This study was about free and forced convective heat transfer in Cu–water Nanofluid past permeable flat vertical semi-infinite moving plate due to high conductivity and occurrence in Cu-water Nanofluid with natural or forced convections in which we consider magnetic field and also heat source.

## 2. Mathematical Formulation

The main assumptions governing the flow and heat transfer are as follows:

- Cu and water nanofluid flow past a moving semi infinite flat plate.
- The nanofluid flow is assumed in the x-direction and initially system is at rest
- This plate is assumed to be non-electrically conducting and moving in the normal direction with the velocity of $U_0$
- Suppose the regular fluid and suspended Cu particle both are at the thermal equilibrium and no slip occur between them.
- Transverse magnetic field is applied uniformly.
- It is assumed that the induced magnetic fieldis less than applied externally.
- The surface temperature at a constant value $T_w$ and the ambient temperature as $T_\infty$ where $T_w > T_\infty$
- The fluid is assumed to follow the Bossinesq approximation

Thus Cartesian co-ordinate system and also the geometry of the plate are shown in the figure.

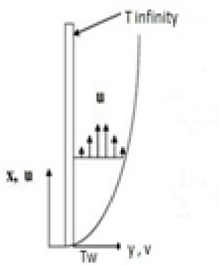


Figure 1: Schematic diagram

## 3. Governing Equations

The boundary layer equations governing the flow and heat transfer as per the assumptions taken in the formulation of the problem are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{1}{\rho_{nf}}\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\mu_{nf}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}\right]+(\rho\beta_T)_{nf}g(T-T_\infty)-\sigma B_0^2 u \quad (2)$$

$$\frac{1}{\rho_{nf}}\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=\mu_{nf}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial x^2}\right] \quad (3)$$

$$\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=\alpha_{nf}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}\right]-\frac{Q}{(\rho c_p)_n f}(T-T_\infty) \quad (4)$$

The boundary conditions for this problem are as follows:
$u(x,y,t)=0, v(x,y,t)=0, T(x,y,t)=T_\infty, t<0, u(0,y,t)=U_0, v(0,y,t)=U_0,$
$T(0,y,t)=T_\infty, y>0, t\geq 0\ u(x,0,t)=U_0, v(x,0,t)=U_0, T(x,0,t)=T_\infty, x>0, t\geq 0$ ,
$u(x,\infty,t)\to 0, v(x,\infty,t)\to 0, T(x,\infty,t)\to 0, t\geq 0$
$u(\infty,y,t)\to 0, v(\infty,y,t)\to 0, T(\infty,y,t)\to 0, t\geq 0$
Here we introduce the following dimensionless variables:

$$X=\frac{x}{L}, Y=\frac{y}{L}, t^{'}=t\frac{U_0}{L}, U=\frac{u}{U_0}, V=\frac{V}{U_0}, \theta=\frac{T-T_\infty}{T_W-T_\infty}. \quad (5)$$

Where L was characteristic length of the plate. The equations (1)-(4) are written in the following dimensionless form using the equations:

$$\left[\frac{\partial U}{\partial t}+u\frac{\partial U}{\partial X}+v\frac{\partial U}{\partial Y}\right]=\frac{1+2.5\mu}{(1-\phi+\phi\frac{\rho_s}{\rho_f})}\frac{1}{Re}\left[\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right]+\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\frac{Gr}{R^2}\theta-M^2U \quad (6)$$

$$\left[\frac{\partial V}{\partial t}+u\frac{\partial V}{\partial X}+v\frac{\partial V}{\partial Y}\right]=\frac{1+2.5\mu}{(1-\phi+\phi\frac{\rho_s}{\rho_f})}\frac{1}{Re}\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right] \quad (7)$$

$$\left[\frac{\partial \theta}{\partial t}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}\right]=\frac{1+4.4Re^{0.4}\phi^{0.66}}{(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f})}\frac{1}{Re}\frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right] \quad (8)$$

## 4. Solution and discussion of the Results

The system of equations from (5)-(8) was coupled. These equations are to be solved when subjected to the boundary conditions. The domain considered is an infinite rectangular plate, the study is in 2 dimensions. For computational purpose the height of the plate is considered to be of 2 units and the width of the plate is to be of 1 unit. To solve this system we used “ND Solve” tool in

Mathematica 10.4. The effect on various parameters were exhibited in graphs from Figures 2 to 9. The profile of every governing parameter is displayed for natural as well as forced convection by considering the Ar>>1 and Ar<<1 respectively. For computation we assumed two cases as Ar = 500 and Ar = 1/500 and the other parameters were assumed to be constant. The vertical velocity (U) profiles are displayed from figures 2 to 5. The vertical flow is found more in natural convection than in forced convection for all variations. Movement of the plate accelerates the flow more in natural convection. Thus the movement of the boundary regulates the flow in forced convection case. From Fig.2 it is evident that the flow was directly proportional to the inertial force in both convections Fig.3 confirms that the solid volume fraction is always inversely proportional to the flow in both convections. But the flow retards gradually.

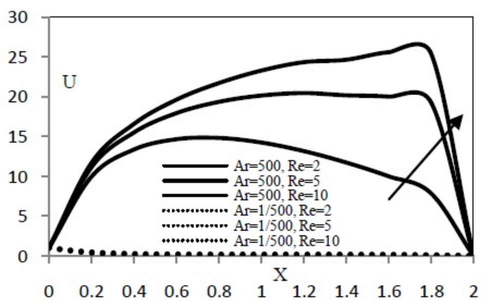


Figure 2: Profiles of U with Re

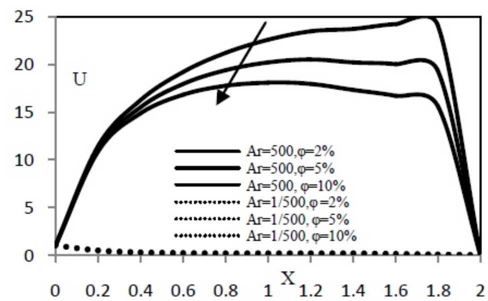


Figure 3: Profiles of velocity U with $\phi$

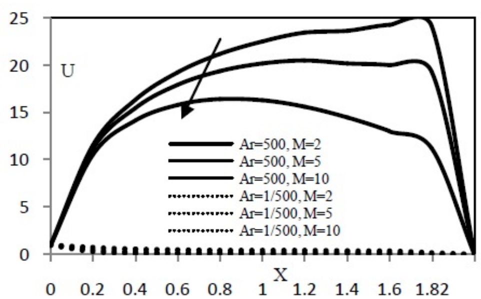


Figure 4: Profiles of velocity U with M

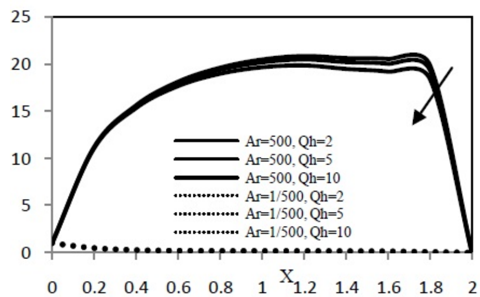


Figure 5: Profiles of U with $Q_h$

From Fig.4 Lorentz force dominates the flow in both convections. Heat source is not much significant on flow in both convections, but it showed negative impact when it was increased (Fig.5). It was happened due to the presence of metal particles which act as heat absorbers. So the Cu-water Nanofluid will be preferred in heat generating systems. The horizontal velocity (V) profiles are displayed from figures 6-9. The horizontal velocity is found more in natural convection than in forced convection for all variations.From Fig.6, viscous force dominates the inertial force and hence the velocity enhances as Re increases in natural convection and reversal effect is observed in forced convection. Fig.7 shows that presences of the metal particle enhance the velocity due to the Brownian motion of the particles. Under further observation, variation of velocity is significant in forced convection than in natural convection. Fig. 8 exhibits the reduction of velocity with Lorentz force in both the convections. But the variation of M is not clearly significant in both convections. Fig.9 shows that variation of v with Qh, heat source is slightly affecting the velocity V. It is found that the variation of Qh reduces the velocity.

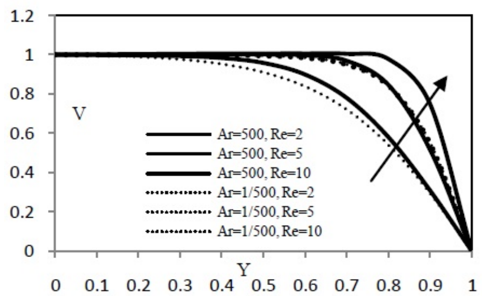


Figure 6: Profile of V with Re

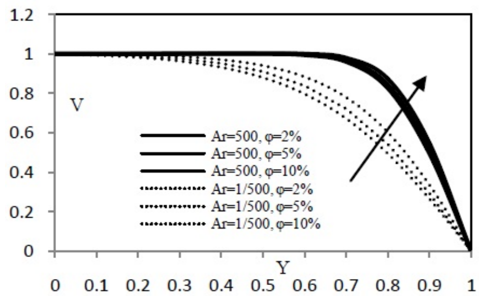


Figure 7: Profile of V with $\phi$

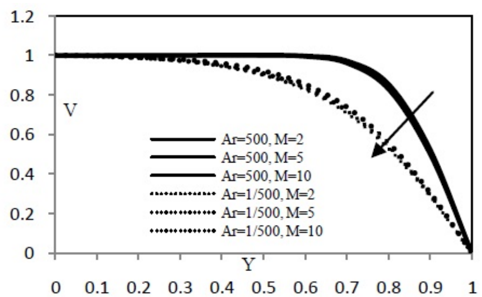


Figure 8: Profile of V with M

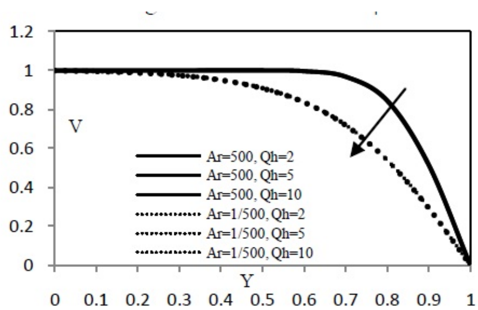


Figure 9: Profile of V with $Q_h$

## 3. Conclusion:

- Rate of heat transfer was found more in forced convection than in the free convection.
- The presence of metal particles enhances the velocity due to Brownian motion of the particles.
- Solid volume fraction was inversely proportional to the temperature in natural convection and it is directly proportional to the temperature in forced convection.

## References

[1] Putra N, Roetzel W, Das SK, Natural convection of nanofluids, Heat and Mass Transfer,2003, vol 39, pp 775–84.

[2] Khanafer K, Vafai K, Light stone M., Buoyancy-driven heat enhancement in a two-dimensional enclosure utilizing nanofluids, International Journal of Heat and Mass Transfer, 2003, vol 46, pp: 3639.

[3] Eastman JA, Choi SUS, Li S, Soyez G, Thompson LJ, DiMelfi RJ, Novel thermal properties of nano structured materials, Materials Science Forum , 1999, Vol 312–314, pp :629–34.

[4] Xuan Y, Li Q, Heat transfer enhancement of nanofluids, International Journal of Heat Fluid Flow, 2000, Vol 21, pp: 58–64.

[5] Pak BC, Cho YI, Hydro dynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles, Experimental Heat Transfer, 1998, Vol 11, pp: 151–70.

[6] Wang X, Xu X, Choi SUS, Thermal conductivity of nanoparticles-fluid mixture, Journal of Thermo physical Heat Transfer, 1999, Vol 13, pp :474– 80.

[7] Neiswanger L, Johnson G.A, Carey V.P, An experimental study of high Rayleigh number mixed convection in a rectangular enclosure with restricted inlet and outlet openings, ASME Journal of Heat Transfer,1987, Vol 109, pp: 446-453.

[8] K. Pushpalath, V. Sugunamma, J. V. R. Reddy, and N. Sandeep, “Heat and Mass Transfer in Unsteady MHD Casson Fluid Flow with Convective Boundary Conditions,” International Journal of Advanced Science and Technology, 1987, vol.91, pp.19–38.

[9] G. Sarojamma, B.Vasundhara, K.Vendabai, MHD Casson Fluid Flow, Heat and Mass Transfer in a Vertical Channel with Stretching Walls, International Journal of Scientific and Innovative Mathematical Research, 2014, Vol 2 (10), PP 800-810.

[10] G. V. P. N. Srikanth, G.Srinivas, V. Ganesh Kumar,,P. Aparna, and B.Suresh Babu, Unsteady Heat and Mass Transfer Flow of Cu-Water Nano-Fluid over an Inclined Plate with Free Convection and Chemical Reactions, International Journal of Pure and Applied Mathematics, 2018, Volume 118, No. 10, 33-46.