ON \(bg^\mu\)-CLOSED MAPS AND \(bg^\mu\)-HOMEOMORPHISMS IN SUPRA TOPOLOGICAL SPACES

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Abstract: The aim of this paper, is to introduce a new class of set namely \(bg^\mu\) - closed maps and \(bg^\mu\) - homeomorphisms in supra topological spaces and study some of their properties. Using these new types of maps, several properties have been obtained.

Keywords and Phrases: \(bg^\mu\) - closed map; \(bg^\mu\) - homeomorphism and \(bg^\mu\) - homeomorphism.

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1. Introduction

In 1983, Mashhour et al [7] introduced the concept of supra topological spaces and studied S- continuous maps and S*- continuous maps. In 2010, Sayed et al [9] introduced and investigated several properties of supra b-open sets and supra b-continuity. In this paper, we introduce the concept of \(bg^\mu\)-closed maps and study its basic properties. Also, we introduce the concept of \(bg^\mu\)-homeomorphisms and investigate several properties for these classes of functions in supra topological
spaces.

2. Preliminaries

Definition 2.1. [7], [9] A subfamily of $\mu$ of $X$ is said to be a supra topology on $X$, if

(i) $X, \emptyset \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\bigcup A_i \in \mu$.

The pair $(X, \mu)$ is called supra topological space. The elements of $\mu$ are called supra open sets in $(X, \mu)$ and complement of a supra open set is called a supra closed set.

Definition 2.2. [7], [9]

(i) The supra closure of a set $A$ is denoted by $\text{cl}_{\mu}(A)$ and is defined as

$$\text{cl}_{\mu}(A) = \bigcap \{B: B \text{ is a supra closed set and } A \subseteq B\}.$$ 

(ii) The supra interior of a set $A$ is denoted by $\text{int}_{\mu}(A)$ and defined as

$$\text{int}_{\mu}(A) = \bigcup \{B: B \text{ is a supra open set and } A \supseteq B\}.$$ 

Definition 2.3. [7] Let $(X, \tau)$ be a topological spaces and $\mu$ be a supra topology on $(X, \tau)$. We call $\mu$ a supra topology associated with $\tau$ if $\tau \subseteq \mu$.

Definition 2.4. [9] Let $(X, \mu)$ be a supra topological space. A set $A$ is called a supra b-open set if $A \subseteq \text{cl}_{\mu}(\text{int}_{\mu}(A)) \cup \text{int}_{\mu}(\text{cl}_{\mu}(A))$. The complement of a supra b-open set is called a supra b-closed set.

Definition 2.5. [8] A subset $A$ of a supra topological space $(X, \mu)$ is called $g^\mu$-closed set if $\text{cl}_{\mu}(A) \subseteq \bigcup$, whenever $A \subseteq \bigcup$ and $\bigcup$ is supra open in $(X, \mu)$.

The complement of $g^\mu$-closed set is called $g^\mu$-open set.

Definition 2.6. A subset $A$ of a supra topological space $(X, \mu)$ is called $bg^\mu$-closed set if $\text{cl}_{\mu}(A) \subseteq \bigcup$, whenever $A \subseteq \bigcup$ and $\bigcup$ is $b^\mu$-open in $(X, \mu)$.

The complement of $bg^\mu$-closed set is called $bg^\mu$-open set.

Definition 2.7. [5] A subset $A$ of a supra topological space $(X, \mu)$ is called $bT^\mu$-closed set if $\text{bcl}_{\mu}(A) \subseteq \bigcup$, whenever $A \subseteq \bigcup$ and $U$ is $T^\mu$-open in $(X, \mu)$. The complement of supra $bT^\mu$-closed set is called $bT^\mu$-open set.

Definition 2.8. [5] Let $(X, \tau)$ and $(Y, \sigma)$ be two supra topological spaces. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called $bT^\mu$-Continuous if $f^{-1}(V)$ is $bT^\mu$-closed in $(X, \mu)$ for every supra closed set $V$ of $(Y, \sigma)$.

Definition 2.9. [3] A subset $A$ of $(X, \mu)$ is called $T^\mu$-closed set if $\text{bcl}_{\mu}(A) \subseteq \bigcup$, whenever $A \subseteq \bigcup$ and $\bigcup$ is $gb^\mu$-open in $(X, \mu)$. The complement of $T^\mu$-closed set is called $T^\mu$-open set.
Definition 2.10. [5] Let \((X,\tau)\) and \((Y,\sigma)\) be two supra topological spaces. A function \(f:(X,\tau)\to (Y,\sigma)\) is called \(bT^\mu\)-irresolute if \(f^{-1}(V)\) is \(bT^\mu\)-closed in \((X,\mu)\) for every \(bT^\mu\)-closed set \(V\) of \((Y,\sigma)\).

Definition 2.11. Let \((X,\tau)\) and \((Y,\sigma)\) be two supra topological spaces. A function \(f:(X,\tau)\to (Y,\sigma)\) is called \(bg^\mu\)-Continuous if \(f^{-1}(V)\) is \(bg^\mu\)-closed in \((X,\mu)\) for every supra closed set \(V\) of \((Y,\sigma)\).

Definition 2.12. Let \((X,\tau)\) and \((Y,\sigma)\) be two supra topological spaces. A function \(f:(X,\tau)\to (Y,\sigma)\) is called \(bg^\mu\)-irresolute if \(f^{-1}(V)\) is \(bg^\mu\)-closed in \((X,\mu)\) for every \(bg^\mu\)-closed set \(V\) of \((Y,\sigma)\).

Definition 2.13. [8] Let \(f:(X,\tau)\to (Y,\sigma)\) where \(\mu\) and \(\lambda\) are supra topological spaces associated with \(\tau\) and \(\sigma\), respectively. Then \(f\) is called supra \(M\)-closed if the image of every supra closed set of \(X\) is supra closed set in \(Y\).

Definition 2.14. [6] A map \(f:(X,\tau)\to (Y,\sigma)\) is said to be \(bT^\mu\)-closed map (\(bT^\mu\)-open map) if the image \(f(A)\) is \(bT^\mu\)-closed (\(bT^\mu\)-open) in \((Y,\sigma)\) for each supra closed (supra open) set \(A\) in \((X,\sigma)\).

Definition 2.15. [6] A bijection \(f:(X,\tau)\to (Y,\sigma)\) is called \(bT^\mu\)-homeomorphism if \(f\) is both \(bT^\mu\)-continuous and \(bT^\mu\)-closed map.

Definition 2.16. [8] A map \(f:(X,\tau)\to (Y,\sigma)\) is said to be \(g^\mu\)-closed map (\(g^\mu\)-open map) if the image \(f(A)\) is \(g^\mu\)-closed (\(g^\mu\)-open) in \((Y,\sigma)\) for each supra closed (supra open) set \(A\) in \((X,\sigma)\).

Definition 2.17. [8] A bijection \(f:(X,\tau)\to (Y,\sigma)\) is called \(g^\mu\)-homeomorphism if \(f\) is both \(g^\mu\)-continuous and \(g^\mu\)-closed map.

3. Basic Properties of \(bg^\mu\) - Closed Maps

Definition 3.1. A map \(f:(X,\tau)\to (Y,\sigma)\) is said to be \(bg^\mu\)-closed map (\(bg^\mu\)-open map) if the image \(f(A)\) is \(bg^\mu\)-closed (\(bg^\mu\)-open) in \((Y,\sigma)\) for each supra closed (supra open) set \(A\) in \((X,\tau)\).

Theorem 3.2. Every supra \(M\)-closed map is \(bg^\mu\)-closed map.

Proof. Let \(f:(X,\tau)\to (Y,\sigma)\) be supra \(M\)-closed map. Let \(V\) be supra closed set in \((X,\tau)\), Since \(f\) is supra \(M\)-closed map then \(f(V)\) is supra closed set in \((Y,\sigma)\). We know that every supra closed set is supra \(bg^\mu\)-closed, then \(f(V)\) is supra \(bg^\mu\)-closed in \((Y,\sigma)\). Therefore \(f\) is supra \(bg^\mu\)-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.3. Let \(X = Y = \{a, b, c\}\), \(\tau = \{X, \phi, \{b\}\}\) and \(\sigma = \{Y, \phi, \{a\}, \{b, c\}\},
The $bg^\mu$-closed sets of $(Y, \sigma)$ are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Let $V = \{a, c\}$ in supra closed set of $(X, \tau)$, $f(V) = f\{a, c\} = \{a, c\}$ is $bg^\mu$-closed in $(Y, \sigma)$ but not supra closed in $(Y, \sigma)$.

Theorem 3.4. Every $bg^\mu$-closed map is $bT^\mu$-closed map. 

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $bT^\mu$-closed map. Let $V$ be supra closed set in $(X, \tau)$. Since $f$ is supra $bg^\mu$-closed map then $f(V)$ is $bT^\mu$-closed set in $(Y, \sigma)$. We know that every $bg^\mu$-closed set is $bT^\mu$-closed, then $f(V)$ is $bT^\mu$-closed in $(Y, \sigma)$. Therefore $f$ is $bT^\mu$-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$. The $bT^\mu$-closed sets are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ and the $bg^\mu$-closed sets are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$. Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Let $V = \{a, c\}$ in supra closed set of $(X, \tau)$, $f(V) = f\{a, c\} = \{a, c\}$ is $bT^\mu$-closed but not $bg^\mu$-closed.

Theorem 3.6. Every $bg^\mu$-closed map is $g^\mu$-closed map.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $g^\mu$-closed map. Let $V$ be supra closed set in $(X, \tau)$, Since $f$ is supra $bg^\mu$-closed map then $f(V)$ is $g^\mu$-closed set in $(Y, \sigma)$. We know that every $bg^\mu$-closed set is $g^\mu$-closed, then $f(V)$ is $g^\mu$-closed in $(Y, \sigma)$. Therefore $f$ is $g^\mu$-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.7. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$. The $g^\mu$-closed sets of $(Y, \sigma)$ are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $bg^\mu$-closed sets of $(Y, \sigma)$ are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Let $V = \{a, c\}$ in supra closed set of $(X, \tau)$, $f(V) = f\{a, c\} = \{a, c\}$ is $g^\mu$-closed but not $bg^\mu$-closed.

Theorem 3.8. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is $bg^\mu$-closed if and only if $bg - cl^\mu f(A) \subseteq f(cl^\mu(A))$ for every subset $A$ of $(X, \tau)$.

Proof. Suppose that $f$ is $bg^\mu$-closed and $A \subseteq X$. Then $f(cl^\mu(A))$ is $bg^\mu$-closed in $(Y, \sigma)$. We have $A \subseteq (cl^\mu(A))$. Thus $f(A) \subseteq cl^\mu(A))$. Then $bg - cl^\mu f(A) \subseteq bg - (cl^\mu f(cl^\mu(A))) = f(cl^\mu(A))$.
Theorem 3.9. A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is \( bg^\mu \)-open if and only if 
\( \text{bg} - \text{int}^\mu(f(A)) \subseteq f(\text{int}^\mu(A)) \) for every subset \( A \) of \( (X, \tau) \).

Proof. Suppose that \( f \) is \( bg^\mu \)-open and \( A \subseteq X \). Then \( f(\text{int}^\mu(A)) \) is \( bg^\mu \)-open in 
\( (Y, \sigma) \). We have \( A \subseteq (\text{int}^\mu(A)) \). Thus \( f(A) \subseteq f(\text{int}^\mu(A)) \). Then 
\( \text{bg} - \text{int}^\mu(f(A)) \subseteq \text{bg} - (\text{int}^\mu(f(\text{int}^\mu(A)))) = f(\text{int}^\mu(A)) \).

Conversely, let \( A \) be any open set in \( (X, \tau) \). Then \( A = \text{int}^\mu(A) \). Thus \( f(A) = f(\text{int}^\mu(A)) \). But \( \text{bg} - \text{int}^\mu(f(A)) \subseteq f(\text{int}^\mu(A)) = f(A) \). Also \( f(A) \subseteq \text{bg} - \text{int}^\mu(f(A)) \).
Thus \( f(A) \) is \( bg^\mu \)-open and hence \( f \) is \( bg^\mu \)-open.

Remark 3.10. The composition of two \( bg^\mu \)-closed maps need not be \( bg^\mu \)-closed map. It is show by the following example

Example 3.11. Let \( X = Y = Z = \{a, b, c\} \). Let \( \tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}, \sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\} \) and \( \eta = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\} \) and \( \text{bg}^\mu \)-closed sets of \( (Y, \sigma) \) are \( \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\} \) and \( \text{bg}^\mu \)-closed sets of \( (Z, \eta) \) are \( \{Z, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\} \). \( f : (X, \tau) \rightarrow (Y, \sigma) \) and 
\( g : (Y, \tau) \rightarrow (Z, \zeta) \) be the identity function. Then \( (gof)\{a, c\} = g(f\{a, c\}) = g(\{a, c\}) = \{a, c\} \) is not supra \( bg^\mu \)-closed map in \( (Z, \zeta) \).

Theorem 3.12. If \( f : X \rightarrow Y \) is a supra closed map and \( g : Y \rightarrow Z \) is \( bg^\mu \)-closed map then the composition \( gof : X \rightarrow Z \) is supra \( bg^\mu \)-closed map.

Proof. Let \( f : X \rightarrow Y \) is a closed map and \( g : Y \rightarrow Z \) is a supra \( bg^\mu \)-closed map. Let 
\( V \) be any supra closed set in \( (X, \tau) \). Since \( f : X \rightarrow Y \) is closed map, \( f(V) \) is closed in 
\( Y \) and since \( g : Y \rightarrow Z \) is supra \( bg^\mu \)-closed map, \( g(f(V)) \) is supra \( bg^\mu \)-closed map in 
\( Z \). This implies \( gof : X \rightarrow Z \) is supra \( bg^\mu \)-closed map.

Remark 3.13. If \( f : X \rightarrow Y \) is a supra \( bg^\mu \)-closed map and \( g : Y \rightarrow Z \) is supra \( M \) closed map then the composition need not be supra \( bg^\mu \)-closed map. It can be seen by the following example.

Example 3.14. Let \( X = Y = Z = \{a, b, c\} \). Let \( \tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}, \sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\} \) and \( \zeta = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\} \) and \( \text{bg}^\mu \)-closed sets of \( (Y, \sigma) \) are \( \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\} \) and \( \text{bg}^\mu \)-closed sets of \( (Z, \zeta) \) are \( \{Z, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\} \). \( f : (X, \tau) \rightarrow (Y, \sigma) \) be the function defined by \( f(a) = a, f(b) = b, f(c) = c \) and \( f : (Y, \tau) \rightarrow (Z, \zeta) \) be the function defined by \( g(a) = a, g(b) = b, g(c) = c \). Here \( f \) is supra \( bg^\mu \)-closed map and \( g \) is supra \( M \) closed map, but its composition is not supra \( bg^\mu \)-closed map, since 
\( gof(\{a, c\}) = \{a, c\} \) is not supra \( bg^\mu \)-closed map in \( (Z, \zeta) \).
Theorem 3.15. For any bijection $f : (X, \tau) \to (Y, \sigma)$ the following are statement are equivalent
(i) $f^{-1} : (Y, \sigma) \to (X, \tau)$ is bg$^\mu$-continuous.
(ii) $f$ is bg$^\mu$-open map.
(iii) $f$ is bg$^\mu$-closed map.

Proof. (i) $\Rightarrow$ (ii) Let $U$ be an supra open set of $(X,\tau)$. By assumption $(f^{-1})^{-1} = f(U)$ is bg$^\mu$-open in $(Y,\sigma)$ and so $f$ is bg$^\mu$-open.
(ii) $\Rightarrow$ (iii) Let $F$ be a supra closed set of $(X,\tau)$. Then $F^c$ is supra open in $(X,\tau)$. By assumption, $f(F^c)$ is bg$^\mu$-open in $(Y,\sigma)$ and therefore $f(F)$ is bg$^\mu$-closed in $(Y,\sigma)$. Hence $f$ is bg$^\mu$-closed.
(iii) $\Rightarrow$ (i) Let $F$ be a supra closed set of $(X,\tau)$. By assumption, $f(F)$ is bg$^\mu$-closed in $(Y,\sigma)$. But $f(F) = (f^{-1})^{-1}(F)$ and therefore $f^{-1}$ is bg$^\mu$-continuous on $(Y,\sigma)$.

4. bg$^\mu$ - Homeomorphism

Definition 4.1. A bijection $f : (X, \tau) \to (Y, \sigma)$ is called bg$^\mu$-homeomorphism if $f$ is both bg$^\mu$-continuous and bg$^\mu$ closed map.

Example 4.2. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$. The bg$^\mu$-closed sets of $(X,\tau)$ are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and bg$^\mu$-closed sets of $(Y,\sigma)$ are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ be the identity maps. Then $f$ is bg$^\mu$-homeomorphism.

Theorem 4.3. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective, bg$^\mu$-continuous map. Then the following are equivalent
(i) $f$ is bg$^\mu$-open map.
(ii) $f$ is bg$^\mu$-homeomorphism.
(iii) $f$ is bg$^\mu$-closed map.

Proof. (i) $\Rightarrow$ (ii) Given $f : (X, \tau) \to (Y, \sigma)$ be a bijective bg$^\mu$-continuous and bg$^\mu$-open. Then by definition, $f$ is an bg$^\mu$-homeomorphism.
(ii) $\Rightarrow$ (iii) Given $f$ is bg$^\mu$-open and bijective. By theorem 3.15(ii), $f$ is a bg$^\mu$-closed map.
(iii) $\Rightarrow$ (i) Given $f$ is bg$^\mu$-closed and bijective. By theorem 3.15(iii), $f$ is a bg$^\mu$-open map.

Remark 4.4. The following example shows that the composition of two bg$^\mu$ - homeomorphism is need not be a bg$^\mu$ - homeomorphism.

Example 4.5. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$ and $\eta = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. The bg$^\mu$-closed sets of $(X,\tau)$ are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}\}$, bg$^\mu$-closed sets of $(Y,\sigma)$
are \{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\} and \(bg^\mu\)-closed sets of \((Z, \eta)\) are \{Z, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}. Then both \(f\) and \(g\) are \(bg^\mu\)-homeomorphism, but their composition \(gof: f : (X, \tau) \rightarrow (Z, \eta)\) is not \(bg^\mu\)-homeomorphism, because for the supra closed set \{a, c\} of \((X, \tau)\) \((gof)\{a, c\} = g(f\{a, c\}) = g(\{a, c\}) = \{a, c\}\), which is not \(bg^\mu\)-closed in \((Z, \eta)\). Therefore \(gof\) is not \(bg^\mu\)-closed and so \(gof\) is not \(bg^\mu\)-homeomorphism.

**Definition 4.6.** A bijection \(f : (X, \tau) \rightarrow (Y, \sigma)\) is called \(*bg^\mu\)-homeomorphism if both \(f\) and \(f^{-1}\) are \(*bg^\mu\)-irresolute.

**Example 4.7** Let \(X = Y = \{a, b, c\}\), \(\tau = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}\) and \(\sigma = \{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}\). The \(bg^\mu\)-closed sets of \((X, \tau)\) are \(\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}\) and \(bg^\mu\)-closed sets of \((Y, \sigma)\) are \(\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}\) and \(\{a, b\}\). Define \(f : (X, \tau) \rightarrow (Y, \sigma)\) be the identity by \(f(a) = a, f(b) = b, f(c) = c\). Then \(f\) is \(*bg^\mu\)-homeomorphism.

**Theorem 4.8.** Every \(*bg^\mu\)-homeomorphism is \(bg^\mu\)-irresolute.

**Proof.** Let \(f\) be a \(*bg^\mu\)-homeomorphism. By the definition of \(*bg^\mu\)-homeomorphism, \(f\) is \(bg^\mu\)-irresolute.

**Remark 4.9.** Every \(bg^\mu\)-irresolute map need not be a \(*bg^\mu\)-homeomorphism.

**Example 4.10.** Let \(X = Y = \{a, b, c\}\), \(\tau = \{X, \phi, \{a\}, \{b\}, \{b, c\}, \{a, c\}\}\) and \(\sigma = \{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}\}\). The \(bg^\mu\)-closed sets of \((X, \tau)\) are \(\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}\) and \(bg^\mu\)-closed sets of \((Y, \sigma)\) are \(\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}\) and \(\{a, b\}\). Define \(f : (X, \tau) \rightarrow (Y, \sigma)\) be the identity by \(f(a) = a, f(b) = c, f(c) = b\). Then \(f\) is \(bg^\mu\)-irresolute, but not \(*bg^\mu\)-homeomorphism. Since \(f(B) = \{a, b\}\), which is not in \(bg^\mu\)-closed in \((Y, \sigma)\).

**References**


