

ON bg^μ -CLOSED MAPS AND bg^μ -HOMEOMORPHISMS IN SUPRA
TOPOLOGICAL SPACES

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Abstract: The aim of this paper, is to introduce a new class of set namely bg^μ - closed maps and bg^μ - homeomorphisms in supra topological spaces and study some of their properties. Using these new types of maps, several properties have been obtained.

Keywords and Phrases: bg^μ - closed map; bg^μ - homeomorphism and $*bg^\mu$ - homeomorphism.

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1. Introduction

In 1983, Mashhour et al [7] introduced the concept of supra topological spaces and studied S- continuous maps and S^* - continuous maps. In 2010, Sayed et al [9] introduced and investigated several properties of supra b-open sets and supra b-continuity. In this paper, we introduce the concept of bg^μ -closed maps and study its basic properties. Also, we introduce the concept of bg^μ -homeomorphisms and investigate several properties for these classes of functions in supra topological

spaces.

2. Preliminaries

Definition 2.1. [7], [9] A subfamily of μ of X is said to be a supra topology on X , if

(i) $X, \phi \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.

The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition 2.2. [7], [9]

(i) The supra closure of a set A is denoted by $cl^\mu(A)$ and is defined as $cl^\mu(A) = \cap \{B: B \text{ is a supra closed set and } A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int^\mu(A)$ and defined as $int^\mu(A) = \cup \{B: B \text{ is a supra open set and } A \supseteq B\}$.

Definition 2.3. [7] Let (X, τ) be a topological spaces and μ be a supra topology on (X, τ) . We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition 2.4. [9] Let (X, μ) be a supra topological space. A set A is called a supra b -open set if $A \subseteq cl^\mu(int^\mu(A)) \cup int^\mu(cl^\mu(A))$. The complement of a supra b -open set is called a supra b -closed set.

Definition 2.5. [8] A subset A of a supra topological space (X, μ) is called g^μ -closed set if $cl^\mu(A) \subseteq \cup$, whenever $A \subseteq \cup$ and \cup is supra open in (X, μ) .

The complement of g^μ -closed set is called g^μ -open set.

Definition 2.6. A subset A of a supra topological space (X, μ) is called bg^μ -closed set if $cl^\mu(A) \subseteq \cup$, whenever $A \subseteq \cup$ and \cup is b^μ -open in (X, μ) .

The complement of bg^μ -closed set is called bg^μ -open set.

Definition 2.7. [5] A subset A of a supra topological space (X, μ) is called bT^μ -closed set if $bcl^\mu(A) \subseteq \cup$, whenever $A \subseteq \cup$ and U is T^μ -open in (X, μ) . The complement of supra bT^μ -closed set is called bT^μ -open set.

Definition 2.8. [5] Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called bT^μ -Continuous if $f^{-1}(V)$ is bT^μ -closed in (X, μ) for every supra closed set V of (Y, σ) .

Definition 2.9. [3] A subset A of (X, μ) is called T^μ -closed set if $bcl^\mu(A) \subseteq \cup$, whenever $A \subseteq \cup$ and \cup is gb^μ -open in (X, μ) . The complement of T^μ -closed set is called T^μ -open set.

Definition 2.10. [5] Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called bT^μ -irresolute if $f^{-1}(V)$ is bT^μ -closed in (X, μ) for every bT^μ -closed set V of (Y, σ) .

Definition 2.11. Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called bg^μ -Continuous if $f^{-1}(V)$ is bg^μ -closed in (X, μ) for every supra closed set V of (Y, σ) .

Definition 2.12. Let (X, τ) and (Y, σ) be two supra topological spaces. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called bg^μ -irresolute if $f^{-1}(V)$ is bg^μ -closed in (X, μ) for every bg^μ -closed set V of (Y, σ) .

Definition 2.13. [8] Let $f : (X, \tau) \rightarrow (Y, \sigma)$ where μ and λ are supra topological spaces associated with τ and σ , respectively. Then f is called supra M -closed if the image of every supra closed set of X is supra closed set in Y .

Definition 2.14. [6] A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be bT^μ -closed map (bT^μ -open map) if the image $f(A)$ is bT^μ -closed (bT^μ -open) in (Y, σ) for each supra closed (supra open) set A in (X, σ) .

Definition 2.15. [6] A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called bT^μ -homeomorphism if f is both bT^μ -continuous and bT^μ closed map.

Definition 2.16. [8] A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be g^μ -closed map (g^μ -open map) if the image $f(A)$ is g^μ -closed (g^μ -open) in (Y, σ) for each supra closed (supra open) set A in (X, σ) .

Definition 2.17. [8] A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called g^μ -homeomorphism if f is both g^μ -continuous and g^μ closed map.

3. Basic Properties of bg^μ - Closed Maps

Definition 3.1. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be bg^μ -closed map (bg^μ -open map) if the image $f(A)$ is bg^μ -closed (bg^μ -open) in (Y, σ) for each supra closed (supra open) set A in (X, τ) .

Theorem 3.2. Every supra M -closed map is bg^μ -closed map.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be supra M -closed map. Let V be supra closed set in (X, τ) , Since f is supra M -closed map then $f(V)$ is supra closed set in (Y, σ) . We know that every supra closed set is supra bg^μ -closed, then $f(V)$ is supra bg^μ -closed in (Y, σ) . Therefore f is supra bg^μ -closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.3. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$,

$\{a, c\}$. The bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = a, f(b) = b, f(c) = c$. Let $V = \{a, c\}$ in supra closed set of (X, τ) , $f(V) = f\{a, c\} = \{a, c\}$ is bg^μ -closed in (Y, σ) but not supra closed in (Y, σ) .

Theorem 3.4. Every bg^μ -closed map is bT^μ -closed map.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bT^μ -closed map. Let V be supra closed set in (X, τ) . Since f is supra bg^μ -closed map then $f(V)$ is bT^μ -closed set in (Y, σ) . We know that every bg^μ -closed set is bT^μ -closed, then $f(V)$ is bT^μ -closed in (Y, σ) . Therefore f is bT^μ -closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$ and $\sigma = \{X, \phi, \{a, c\}, \{c, d\}\}$. The bT^μ -closed sets are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$ and the bg^μ -closed sets are $\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{c, d\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by

$f(a) = a, f(b) = b, f(c) = c$. Let $V = \{a, c\}$ in supra closed set of (X, τ) , $f(V) = f\{a, c\} = \{a, c\}$ is bT^μ -closed but not bg^μ -closed.

Theorem 3.6. Every bg^μ -closed map is g^μ -closed map.

Proof. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a g^μ -closed map. Let V be supra closed set in (X, τ) , Since f is supra bg^μ -closed map then $f(V)$ is g^μ -closed set in (Y, σ) . We know that every bg^μ -closed set is g^μ -closed, then $f(V)$ is g^μ -closed in (Y, σ) . Therefore f is g^μ -closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.7. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, b\}, \{b, c\}\}$. The g^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ and bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{c\}, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by

$f(a) = a, f(b) = b, f(c) = c$. Let $V = \{a, c\}$ in supra closed set of (X, τ) , $f(V) = f\{a, c\} = \{a, c\}$ is g^μ -closed but not bg^μ -closed.

Theorem 3.8. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is bg^μ -closed if and only if $bg - cl^\mu f(A) \subseteq f(cl^\mu(A))$ for every subset A of (X, τ) .

Proof. Suppose that f is bg^μ -closed and $A \subseteq X$. Then $f(cl^\mu(A))$ is bg^μ -closed in (Y, σ) . We have $A \subseteq (cl^\mu(A))$. Thus $f(A) \subseteq f(cl^\mu(A))$. Then $bg - cl^\mu f(A) \subseteq bg - (cl^\mu f(cl^\mu(A))) = f(cl^\mu(A))$.

Conversely, let A be any closed set in (X, τ) . Then $A = cl^\mu(A)$. Thus $f(A) = f(cl^\mu(A))$. But $bg - cl^\mu f(A) \subseteq f(cl^\mu(A)) = f(A)$. Also $f(A) \subseteq bg - cl^\mu(f(A))$. Thus $f(A)$ is bg^μ -closed and hence f is bg^μ -closed.

Theorem 3.9. *A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is bg^μ -open if and only if $bg - int^\mu f(A) \subseteq f(int^\mu(A))$ for every subset A of (X, τ) .*

Proof. Suppose that f is bg^μ -open and $A \subseteq X$. Then $f(int^\mu(A))$ is bg^μ -open in (Y, σ) . We have $A \subseteq (int^\mu(A))$. Thus $f(A) \subseteq f(int^\mu(A))$. Then $bg - int^\mu f(A) \subseteq bg - (int^\mu f(int^\mu(A))) = f(int^\mu(A))$.

Conversely, let A be any open set in (X, τ) . Then $A = int^\mu(A)$. Thus $f(A) = f(int^\mu(A))$. But $bg - int^\mu f(A) \subseteq f(int^\mu(A)) = f(A)$. Also $f(A) \subseteq bg - int^\mu(f(A))$. Thus $f(A)$ is bg^μ -open and hence f is bg^μ -open.

Remark 3.10. *The composition of two bg^μ -closed maps need not be bg^μ -closed map. It is show by the following example*

Example 3.11. *Let $X = Y = Z = \{a, b, c\}$. Let $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$ and $\eta = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. The bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ and bg^μ -closed sets of (Z, η) are $\{Z, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \tau) \rightarrow (Z, \zeta)$ be the identity function. Then $(gof) \{a, c\} = g(f \{a, c\}) = g(\{a, c\}) = \{a, c\}$ is not supra bg^μ -closed map in (Z, ζ) .*

Theorem 3.12. *If $f : X \rightarrow Y$ is a supra closed map and $g : Y \rightarrow Z$ is bg^μ -closed map then the composition $gof : X \rightarrow Z$ is supra bg^μ -closed map.*

Proof. Let $f : X \rightarrow Y$ is a closed map and $g : Y \rightarrow Z$ is a supra bg^μ -closed map. Let V be any supra closed set in (X, τ) . Since $f : X \rightarrow Y$ is closed map, $f(V)$ is closed in Y and since $g : Y \rightarrow Z$ is supra bg^μ -closed map, $g(f(V))$ is supra bg^μ -closed map in Z . This implies $gof : X \rightarrow Z$ is supra bg^μ -closed map.

Remark 3.13. *If $f : X \rightarrow Y$ is a supra bg^μ -closed map and $g : Y \rightarrow Z$ is supra M closed map then the composition need not be supra bg^μ -closed map. It can be seen by the following example.*

Example 3.14. *Let $X = Y = Z = \{a, b, c\}$. Let $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$ and $\zeta = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. The bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ and bg^μ -closed sets of (Z, ζ) are $\{Z, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. $f : (X, \tau) \rightarrow (Y, \sigma)$ be the function defined by $f(a) = a, f(b) = b, f(c) = c$ and $g : (Y, \tau) \rightarrow (Z, \zeta)$ be the function defined by $g(a) = a, g(b) = b, g(c) = c$. Here f is supra bg^μ -closed map and g is supra M closed map, but its composition is not supra bg^μ -closed map, since $gof(\{a, c\}) = \{a, c\}$ is not supra bg^μ -closed map in (Z, ζ) .*

Theorem 3.15. For any bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ the following are statement are equivalent

(i) $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is bg^μ -continuous.

(ii) f is bg^μ -open map.

(iii) f is bg^μ -closed map.

Proof. (i) \Rightarrow (ii) Let U be an supra open set of (X, τ) . By assumption $(f^{-1})^{-1} = f(U)$ is bg^μ -open in (Y, σ) and so f is bg^μ -open.

(ii) \Rightarrow (iii) Let F be a supra closed set of (X, τ) . Then F^c is supra open in (X, τ) . By assumption, $f(F^c)$ is bg^μ -open in (Y, σ) and therefore $f(F)$ is bg^μ -closed in (Y, σ) . Hence f is bg^μ -closed.

(iii) \Rightarrow (i) Let F be a supra closed set of (X, τ) . By assumption, $f(F)$ is bg^μ - closed in (Y, σ) . But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is bg^μ -continuous on (Y, σ) .

4. bg^μ - Homeomorphism

Definition 4.1. A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called bg^μ -homeomorphism if f is both bg^μ -continuous and bg^μ closed map.

Example 4.2. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$. The bg^μ -closed sets of (X, τ) are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity maps. Then f is bg^μ -homeomorphism.

Theorem 4.3. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective, bg^μ -continuous map. Then the following are equivalent

(i) f is bg^μ -open map.

(ii) f is bg^μ -homeomorphism.

(iii) f is bg^μ -closed map.

Proof. (i) \Rightarrow (ii) Given $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective bg^μ -continuous and bg^μ -open. Then by definition, f is an bg^μ -homeomorphism.

(ii) \Rightarrow (iii) Given f is bg^μ -open and bijective. By theorem 3.15(ii), f is a bg^μ -closed map.

(iii) \Rightarrow (i) Given f is bg^μ -closed and bijective. By theorem 3.15(iii), f is a bg^μ -open map.

Remark 4.4. The following example shows that the composition of two bg^μ - homeomorphism is need not be a bg^μ - homeomorphism.

Example 4.5. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$ and $\eta = \{Z, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. The bg^μ -closed sets of (X, τ) are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}\}$, bg^μ -closed sets of (Y, σ)

are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$ and bg^μ -closed sets of (Z, η) are $\{Z, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. Then both f and g are bg^μ -homeomorphism, but their composition $gof: f:(X, \tau) \rightarrow (Z, \eta)$ is not bg^μ -homeomorphism, because for the supra closed set $\{a, c\}$ of (X, τ) $(gof)\{a, c\} = g(f\{a, c\}) = g(\{a, c\}) = \{a, c\}$, which is not bg^μ -closed in (Z, η) . Therefore gof is not bg^μ -closed and so gof is not bg^μ -homeomorphism.

Definition 4.6. A bijection $f:(X, \tau) \rightarrow (Y, \sigma)$ is called $*bg^\mu$ -homeomorphism if both f and f^{-1} are bg^μ -irresolute.

Example 4.7 Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$. The bg^μ -closed sets of (X, τ) are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Define $f:(X, \tau) \rightarrow (Y, \sigma)$ be the identity by $f(a) = a, f(b) = b, f(c) = c$. Then f is $*bg^\mu$ -homeomorphism.

Theorem 4.8. Every $*bg^\mu$ -homeomorphism is bg^μ -irresolute.

Proof. Let f be a $*bg^\mu$ -homeomorphism. By the definition of $*bg^\mu$ -homeomorphism, f is bg^μ -irresolute.

Remark 4.9. Every bg^μ -irresolute map need not be a $*bg^\mu$ -homeomorphism.

Example 4.10. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$. The bg^μ -closed sets of (X, τ) are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and bg^μ -closed sets of (Y, σ) are $\{Y, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Define $f:(X, \tau) \rightarrow (Y, \sigma)$ be the identity by $f(a) = a, f(b) = c, f(c) = b$. Then f is bg^μ -irresolute, but not $*bg^\mu$ -homeomorphism. Since $f(\{a, c\}) = \{a, b\}$ which is not in bg^μ -closed in (Y, σ) .

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