

Fractional Integral Transformations of Mittag-Leffler Type E -function

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Abstract : This paper deals with various fractional integral transformations of Mittag-Leffler type E -function and obtain results for earlier defined Mittag-Leffler type functions.

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1. Introduction

By recent works of several authors, it has proved that Mittag-Leffler (M-L) function is the solution of fractional differential and integral equations. Many authors have denned various generalizations of M-L function. In an effort to unify results of various forms of M-L function we have defined a unified M-L type function named S-function [1]. Here we study Erdélyi-Kober, Riemann-Liouville and other fractional integral transformation of newly defined M-L type E -function.

Throughout this paper, we use the following definitions

- Riemann-Liouville fractional integral operator $(I_{c+}^{\theta} \Psi)(x)$ [5]

$$(I_{c+}^{\theta} \Psi)(x) = \frac{1}{\Gamma(\theta)} \int_c^x (x-t)^{\theta-1} \Psi(t) dt \quad (1)$$

where $\theta \in \mathbb{C}$ and $\Re(\theta) > 0$.

- Erdélyi-Kober fractional integral operator $(\Xi_{o+}^{\eta, \theta} f)(x)$ [5]

$$(\Xi_{o+}^{\eta, \theta} f)(x) = \frac{x^{-\eta-\theta}}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} t^{\theta} f(t) dt \quad (2)$$

where $\eta, \theta \in \mathbb{C}$; $\Re(\eta) > 0$ and $\Re(\theta) > 0$.

- In 1903, Gösta Mittag-Leffler [4], introduced the function $E_{\alpha}(z)$, defined as

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(\alpha n + 1)} z^n \quad (3)$$

where $z, \alpha \in \mathbb{C}$; $\Re(\alpha) \geq 0$ and $|z| < \infty$.