South East Asian J. of Math. & Math. Sci. **ISSN : 0972-7752**
Vol.11, No.1 2015, pp. 71-78

On Certain continued Fractions

B.P. Mishra, Ashutosh Singh* and Amit Kumar Singh*
Department of Mathematics,
M.D. College, Parel, Mumbai, India
*Department of Mathematics,
T.D.P.G. College, Jaunpur, U.P., India

Abstract: In this paper, we establish two equivalent continued fractions and deduce certain interesting special cases of the same.
Keywords and phrases: Hypergeometric functions, q-series, continued fraction.
2000 A.M.S. subject classification: Primary 33D15, secondary 33F10.

1. Introduction

Hypergeometric functions have very intimate relationship with continued fraction. Such relations occupy prime position in the work of Srinivasa Ramanujan. Motivated by a paper of Andrews et. al [1], we establish certain results involving q-series and continued fractions which are of general nature.

2. Notations and Definitions:

With α and q ($|q| < 1$), real or complex we write

$$(\alpha;q)_n = \begin{cases} (1-\alpha)(1-\alpha q)(1-\alpha q^2)\dots(1-\alpha q^{n-1}) & \text{if } n>0; \\ 1 & \text{if } n=0. \end{cases}$$

and

$$(\alpha;q)_\infty = \prod_{r=0}^{\infty}(1-\alpha q^r).$$

A continued fraction is denoted by

$$a_0 + \frac{b_1}{a_1+}\frac{b_2}{a_2+}\frac{b_3}{a_3+}\dots\frac{b_n}{a_n}.$$

If $n \to \infty$, it becomes infinite continued fraction.

3. Main Results

In this section we shall establish the following results.

If $\Phi(x,\alpha) = \sum_{n=0}^{\infty}\frac{x^n q^{n(n+1)/2}}{(q;q)_n(\alpha;q)_n}$, then

$$(1-\alpha)\frac{\Phi(x,\alpha)}{\Phi(xq,\alpha q)} = (1+xq-\alpha) + \frac{x\alpha q^2}{(1-\alpha q+xq^2)+}\frac{x\alpha q^4}{(1-\alpha q^2+xq^3)+}$$