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Certain Results Involving Lambert Series and Continued Fractions-I

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Abstract: In this paper, an attempt has been made to establish certain results involving Lambert series and continued fractions.
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1. Introduction, Notations and Definitions

Here a and in the sequel we employ the customary notations for $|q| < 1$,

$$[a;q]_0 = 1, \text{and for } n \geq 1, \text{let}$$

$$[a;q]_n = (1-a)(1-aq)(1-aq^2)...(1-aq^{n-1})$$

and

$$[a;q]_\infty = \prod_{r=0}^{\infty}(1-aq^r).$$

Also,

$$[a_1, a_2, a_3, ..., a_r; q]_n = [a_1;q]_n[a_2;q]_n...[a_r;q]_n.$$

The celebrated ${}_1\Psi_1$ summation formula [4, p. 196, Entry 17] of Ramanujan can be stated as,

$$\sum_{k=1}^{\infty}\frac{[1/\alpha;q]_k(-\alpha q^{1/2})^k z^k}{[\beta;q]_k} + \sum_{n=0}^{\infty}\frac{[q/\beta;q]_k(-\beta q^{-1/2})^k z^{-k}}{[\alpha q;q]_k}$$

$$= \frac{[-zq^{1/2};q]_\infty[-q^{1/2}/z;q]_\infty[q;q]_\infty[\alpha\beta;q]_\infty}{[-\alpha zq^{1/2};q]_\infty[-\beta/zq^{1/2};q]_\infty[\alpha q;q]_\infty[\beta;q]_\infty} \quad (1.1)$$

Differentiating (1.1) with respect to z and then setting $z = -q^{-1/2}$, Bhargava and Somashekara [2] established the identity,

$$\sum_{k=0}^{\infty}\frac{(k+1)[q/\alpha;q]_k\alpha^k}{[\beta;q]_{k+1}} + \sum_{k=0}^{\infty}\frac{k[q/\beta;q]_k\beta^k}{[\alpha;q]_{k+1}} = \frac{[q;q]_\infty^3[\alpha\beta;q]_\infty}{[\alpha;q]_\infty^2[\beta;q]_\infty^2}. \quad (1.2)$$