

Basic Hypergeometric Series and Fourier Transform

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Abstract: In this paper, we shall make use of Bailey's transform and Fourier transform, in order to establish summations formulae for basic hypergeometric series.

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1. Introduction, Notations and Definitions

We employ the usual notations

$$[a; q]_n = (1 - a)(1 - aq) \dots (1 - aq^{n-1}), \quad n = 1, 2, 3, \dots,$$

$$[a; q]_0 = 1,$$

$$[a_1, a_2, \dots, a_r; q]_n = [a_1; q]_n [a_2; q]_n \dots [a_r; q]_n.$$

and for $|q| < 1$,

$$[a; q]_\infty = \prod_{r=0}^{\infty} (1 - aq^r)$$

$$[a_1, a_2, \dots, a_r; q]_\infty = [a_1; q]_\infty [a_2; q]_\infty \dots [a_r; q]_\infty.$$

The basic hypergeometric series is defined as,

$${}_r\Phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q; z \\ b_1, b_2, \dots, b_s \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n z^n}{(q, b_1, b_2, \dots, b_s; q)_n}, \quad (1.1)$$

where for convergence $(|z|, |q|) < 1$, if $1 + s = r$ and for $1 + s > r$, $|z| < \infty$.

W.N. Bailey [1] in 1944 established a simple but a very useful transform called as Bailey transform: if

$$\beta_n = \sum_{r=0}^n \alpha_r u_{n-r} v_{n+r}$$