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On the Projective Flatness of a Finsler space with infinite series (α, β)-metric $\frac{\beta^2}{\beta-\alpha}$

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Abstract: The (α, β)-metric is a Finsler metric which is constructed from a Riemannian metric α and a differential 1-form β; it has been sometimes treated in theoritical physics. We consider the projective flatness of Finsler space with (α, β)-metric. Projective Finsler geometry is an important part of Finsler geometry and has been studied by so many differential geometers e.g., Matsumoto, Shen, H. S. Park etc. For two Finsler spaces (M, F) and $(M, \bar{F})$ on a common underlying manifold M, we say (M, F) is projective to $(M, \bar{F})$ (or Finsler metric F is projectively related to Finsler metric $\bar{F}$) if any geodesic of (M, F) coincides with a geodesic of $(M, \bar{F})$ as a set of points and vice-versa. The purpose of this paper, is to find the conditions for Finsler space with infinite series (α, β)-metric -$\frac{\beta^2}{\beta-\alpha}$ to be projectively flat.

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1. Introduction

Let $F^n = (M^n, L)$ be an n-dimensional Finsler space, that is, an n-dimensional differentiable manifold M^n equipped with a fundamental function L(x,y). The concept of an (α, β)- metric $L(\alpha, \beta)$ was introduced in 1972 by M. Matsumoto [6]. A Finsler metric L(x,y) is called an (α, β)-metric $L(\alpha, \beta)$, if L is positively homogenous function of α and β of degree one, where $\alpha^2 = a_{ij}(x)y^iy^j$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a one form on M^n. The most interesting examples of (α, β) metric are the Randers metric, Kropina metric and Matsumoto metric.

A Finsler space $F^n = (M^n, L)$ is called a locally Minkowski space[8], if M^n is covered by coordinate neighborhood system (x^i) in each of which L is a function of y^i only and the geodesics can be represented by $n-1$ linear equations of coordinates. A Finsler space $F^n = (M^n, L)$ is called projectively flat, if F^n is projective to a