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Three Expansions for a Three Variable Hypergeometric Function

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Abstract: In this paper we record three summation results for a triple hypergeometric series X_2 and discuss various cases of reducibility.
Keywords: Hypergeometric function, Horn function, Appell function, Laguerre polynomial, Jacobi polynomial.

1. Introduction:

Exton [1] introduced a triple hypergeometric series whose representation is

$$X_2(a,b;c_1,c_2,c_3;x,y,z)=\sum_{0}^{\infty}\frac{(a)_{2m+2n+p}(b)_p}{(c_1)_m(c_2)_n(c_3)_p}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!} \tag{1.1}$$

where

$$(a)_m=\frac{\Gamma(a+m)}{\Gamma(a)}, \qquad a=0,-1,-2,...$$

The precise three dimensional region of convergence of (1.1) is given by, see [2],

$$2\sqrt{r}+2\sqrt{s}+t \quad 1, |x| \quad r, |y| \quad s, \text{ and } \quad |z| \quad t$$

where the positive quantities r, s, and t are associated radii of convergence. For details of this function and other many related series refer to Exton [1] and Srivastava and Karlsson [3].

The Laplace type integral representation of (1.1.) due to Exton is

$$X_2(a,b;c_1,c_2,c_3;x,y,z)$$
$$=\frac{1}{\Gamma(a)}\int_0^{\infty}e^{-s}s^{a-1}\ {}_0F_1(-;c_1;xs^2)\ {}_0F_1(-;c_2;ys^2)\ {}_1F_1(b;c_3;zs)ds \tag{1.2}$$

where $Re(a)>0$.

2. In this section we derive the following,

$$\sum_{m=0}^{n}(-1)^m\begin{pmatrix} n \\ m \end{pmatrix}\frac{(\alpha+n)_m}{(1+\alpha)_m}X_2(a,m-n;c_1,c_2,1+\alpha+m;x,y,z)$$