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DECIMAL EXPANSIONS: THEIR UNIQUENESS AND INTERESTING PATTERNS

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Dedicated to Prof. K. Srinivasa Rao on his 75^{th} ***Birth Anniversary***

Abstract: An expression of the form o.abcd... written by arbitrarily picking up a, b, c, d ... from among 0 to 9, cannot be taken to represent the decimal expansion of a real number. In fact, the decimal expansion of every irrational number is unique in itself whereas those of rational numbers (fractions) follow some interesting patterns. These results are of far-reaching consequences in the context of Cantor's second proof (1878) about the uncountability of real numbers.

Keywords and Phrases: Decimal expansion, Interesting patterns.

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1. Introduction

Representation of rational numbers (fractions) and irrational numbers in decimal system facilitate in arithmetical operations. It is well-known that decimal expansion of every irrational number is unique in itself [1]. But, herein it has been shown that the decimal expansions of rational numbers (fractions) have some very interesting patterns. These results are of far reaching consequences in the context of Cantor's second proof (1878) about the uncountability of real numbers [2].

2. Some interesting patterns in the decimal expansions of fractions

Fractions of the form a/b, subject to $1 \leq a \leq b-1$ have either terminating decimal expansions (tde) or non-tde. In tde, the digits to the right of the decimal point are finite e.g. $\frac{1}{2} = 0.5$. But, in non-tde, the digits to the right of the decimal point are infinite but repeating themselves after finite number of digits, called its period P e.g. $1/3 = 0.333...$, $1/11 = 0.0909....$, $1/37 = 0.027027....$ have $P = 1, 2, 3$ respectively. In general, if p is a prime then P of $1/p$ is given by $P = (p-1)/j$