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HIERARCHIES OF PALINDROMIC SEQUENCES IN THE SYMMETRIC GROUP S_n

K. Srinivasa Rao and Pankaj Pundir*

Visiting Professor, Indian Institute of Information Technology,
Chittoor, Andhra Pradesh, 517588, INDIA
& Senior Professor (Retd.), Institute of Mathematical Sciences,
Taramani, Chennai-600113, India
E-mail: ksrao18@gmail.com

*Indian Institute of Information Technology,
Chittoor, Andhra Pradesh, 517588, INDIA
E-mail: pundir.pankaj25@gmail.com

Dedicated to Prof. K. Srinivasa Rao on his 75^{th} ***Birth Anniversary***

Abstract: A new property of the Symmetric group, S_n, arises when each element is assigned a unique place value, which enables the ordering of the elements (numerically). It is shown that the differences between successive elements of this ordered, place-value assigned symmetric group S_n, give rise to a palindromic sequence $(\mathcal{P}_k)^{\beta}_{1\le k\le n!-1}$. We define the family of palindromic sequences, associated to S_n. Sieving out a given number at a time in the maximal palindromic sequence of the group, S_n, of length $n!-1$, results in a hierarchy of palindromic sequences, ending with a single element, which will be the central element of $(\mathcal{P}_k)^{\beta}_{1\le k\le n!-1}$. Consequences of the concept of place value ordering of the elements of S_n, are presented in this article.

1. Introduction

Symmetric groups have been extensively studied in the field of abstract Algebra. A symmetric group is the set of all the permutations of the indices $\{1, 2, \ldots, n\}$, denoted by S_n. As the number of permutations of indices $\{1, 2, \ldots, n\}$ are $n!$, S_n is a finite group of order $n!$.

Definition 1. Let $<$ be a lexicographic ordering on the elements of S_n. Consider the two permutations of S_n, $\sigma = (a_1\ a_2\ \cdots\ a_n)$ and $\pi = (b_1\ b_2\ \cdots\ b_n)$. We say $\sigma < \pi$, if there exists an element $i \in [n]$, such that $\forall\ j < i$, $\sigma_{(j)} = \pi_{(i)}$ and