

A VARIANT OF THE TYPE-1 BETA AND DIRICHLET INTEGRALS

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Dedicated to Prof. K. Srinivasa Rao on his 75th Birth Anniversary

Abstract: A double integral, which can be taken as an extension of a type-1 beta integral, is introduced and its properties are studied. Then a k -variate multiple integral on a unit cube in Euclidean k -space is introduced, which will produce the total integral equivalent to the total integral in a $(k - 1)$ -variate type-1 Dirichlet integral, or the integral over a simplex in a $(k - 1)$ -flat. Several properties of this multiple integral are studied. Marginal functions and Mellin transform are examined. Several transformations are given, finally leading to a type-1 Dirichlet integral. Statistical densities connected with the integrand of the extended Dirichlet integral are also discussed.

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1. Introduction

Type-1 beta integral and beta function are very important in many areas. Type-1 beta model is a popular prior probability measure in Bayesian analysis. Variation in the chance of occurrence of any event is usually modeled by a type-1 beta model. In population studies, the probability of conception is usually modeled by a type-1 beta model. In random cuts or random division of an interval, a type-1 beta model