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SOLUTIONS OF PARAMETRIC QUADRATIC DIOPHANTINE EQUATIONS OVER $\mathbb{Z}$ AND $\mathbb{F}_p$

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Abstract: In this paper, we find the integer solutions to the Diophantine equation $D : x^2 - (a^2b^2c^2 - ab)y^2 - (4ab - 2)x + (4a^2b^2c^2 - 4ab)y + 4(a^2b^2 - a^2b^2c^2) = 0$ over $\mathbb{Z}$ and finite fields $\mathbb{F}_p$ for a, b and c are integers and primes $p \geq 2$, respectively. Moreover, We find positive integer solutions of the equation D in terms of generalized Fibonacci and Lucas sequences.

Keywords and Phrases: Diophantine Equation, Pell Equation, Continued Fractions, Finite fields, Fibonacci and Lucas sequences.

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1. Introduction

We call a Diophantine equation an equation of the form

$$f(x_1, x_2, ..., x_n) = 0 \tag{1.1}$$

Where f is an n-variable function with $n \geq 2$. If f is a polynomial with integral coefficients, then (1.1) is an algebraic Diophantine equation. An n-tuple $(x_1^0, x_2^0, ..., x_n^0) \in \mathbb{Z}^n$ satisfying (1.1) is called a solution of the equation. An equation having one or more solutions is called solvable (see[1]). Diophantus's work on equations of type (1.1) was continued by Chinese mathematicians, Arabs and taken to a deeper level by Fermat, Euler, Lagrange, Gauss, and many others. This topic remains of great importance in contemporary mathematics (see[2],[3]). There is no universal method for determining whether a Diophantine equation has a solution, or for finding them all if solutions exists. However, we are quite successful in dealing with polynomials of low degree, or in a small number of variables (see[10]).