

GROUP THEORETICAL ASPECTS OF GENERALIZED HYPERGEOMETRIC FUNCTIONS

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Dedicated to Prof. M.A. Pathan on his 75th birth anniversary

Abstract: In this survey article, the group theory of the 24 Kummer solutions of the Gauss second order ordinary differential equation and the group theory of the ${}_3F_2(a, b, c; d, e; 1)$ transformations of Weber-Erdelyi, giving rise to a new 72-element group.

1. Introduction

Leonhard Euler (1707 - 1783) is perhaps the first to study the hypergeometric functions, in 1748. The modern framework for the hypergeometric series and the corresponding hypergeometric functions is due to Gauss. Carl Friedrich Gauss (1777 - 1855), the German mathematician has been acknowledged as one of the three leading mathematicians of all time, with Archimedes (287 B.C. – 214/212 B.C.) and Sir Isaac Newton (1642 – 1727), being the other two. Besides his contribution to the theory of numbers, his outstanding work includes the discovery of the Method of Least Squares, the hypergeometric series and non-Euclidian geometry. His collected papers run to several volumes and were being edited at Göttingen in the 20th century. In 1812, recognizing the importance of the property of convergence of an infinite series, he published his comprehensive thesis on “*Disquisitiones generales circa seriem infinitam*” [1]. Historically, the geometric series:

$$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{k=0}^{\infty} x^k, \quad \forall 0 \leq x < 1, \quad (1)$$

the first theorem which one comes across in School as a special case of the Binomial