# Controlled $K$-frames in Hilbert Spaces

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*Dedicated to Prof. Hari M. Srivastava on his $75^{th}$ birth anniversary*

**Abstract:** $K$-frames were recently introduced by L. Găvruta in Hilbert spaces to study atomic systems with respect to bounded linear operator. Also controlled frames have been recently introduced by P. Balazs in Hilbert spaces to improve the numerical efficiency of interactive algorithms for inverting the frame operator. In this manuscript, we will define the concept of the controlled $K$-frames and will show that controlled $K$-frames are equivalent to $K$-frames and so the controlled operator $C$ can be used as preconditions in applications.

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## 1. INTRODUCTION

Frames in Hilbert spaces were first proposed by Duffin and Schaeffer to deal with nonharmonic Fourier series in 1952 [8] and widely studied from 1986 since the great work by Daubechies at al.[9]. Now frames play an important role not only in the theoretics but also in many kinds of applications and have been widely applied in signal processing [12], sampling [10, 11], coding and communications [16], filter bank theory [3], system modeling [7] and so on. For special applications many other types of frames were proposed, such as the fusion frames [4, 5] to deal with hierarchical data processing, $g$-frames [17] by Sun to deal with all existing frames as united object, oblique dual frames [10] by Elder to deal with sampling reconstructions, and so on.

The notion of $K$-frames was recently introduced by L. Găvruta to study the atomic systems with respect to a bounded linear operator $K$ in Hilbert spaces. From [18], we know that $K$-frames are more general than ordinary frames in sense that the lower frame bound only holds for the elements in the range of the $K$, where $K$ is a bounded linear operator in a separable Hilbert Space $H$.