J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.4, No.1 (2015), pp. 65-70

A note on confluent hypergeometric series

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Abstract: In this paper, using second order differential equation and some contiguous relations of ordinary hypergeometric series we have established Kummer's transformations for confluent hypergeometric series.
Keywords and Phrases: Ordinary hypergeometric series, Identities and transformation formulae .
Mathematics Subject Classification: Primary 33D15, 33D90, 11A55; Secondary 11F20.

1. Introduction, Notations and Definitions

We know that the second order differential equation

$$z(1-z)\frac{d^2w}{dz^2}+\{c-(1+a+b)z\}\frac{dw}{dz}-abw=0 \tag{1.1}$$

has three regular singularities at 0, 1 and ∞. If we put z/b for z in (1.1) we get the new form of this equation as,

$$b^2\frac{z}{b}\left(1-\frac{z}{b}\right)\frac{d^2w}{dz^2}+\left\{c-(1+a+b)\frac{z}{b}\right\}b\frac{dw}{dz}-abw=0$$

which on simplification gives,

$$z\left(1-\frac{z}{b}\right)\frac{d^2w}{dz^2}+\left\{c-(1+a+b)\frac{z}{b}\right\}\frac{dw}{dz}-aw=0. \tag{1.2}$$

It has three regular singularities at 0, b, ∞.
Now, let $b\to\infty$ in (1.2), then it takes the form

$$z\frac{d^2w}{dz^2}+(c-z)\frac{dw}{dz}-aw=0. \tag{1.3}$$

In (1.3), there is the confluence of two singularities, so it is called confluent hypergeometric equation.