

Certain Explicit Evaluation of Ramanujan's Theta Functions

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Abstract: In this paper, using certain known modular equations, we have established certain modular identities which have been made use of, to evaluate certain theta functions. We shall attempt to evaluate Ramanujan's cubic continued fraction and also certain Ramanujan-Weber class invariants with the help of some of our results. The results established herein may prove useful in further investigations in the subject.

Keywords and Phrases: Modular identities, modular equation, Ramanujan's theta function, cubic continued fraction, Ramanujan-Weber class invariants.

2000 AMS subject classification: 01A60, 01A75, 33A30.

1. Introduction, Notations and Definitions : In what follows, for real or complex α and q ($|q| < 1$), let

$$[\alpha; q]_{\infty} = \prod_{r=0}^{\infty} (1 - \alpha q^r). \quad (1.1)$$

Ramanujan's theta functions are defined by

$$\phi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = [q^2; q^2]_{\infty} [-q; q^2]_{\infty}^2 \quad (1.2)$$

$$\psi(q) = \sum_{n=0}^{\infty} q^{n(n+1)/2} = \frac{[q^2; q^2]_{\infty}}{[q; q^2]_{\infty}} \quad (1.3)$$

$$f(-q) = \sum_{n=-\infty}^{\infty} (-)^n q^{n(3n-1)/2} = [q; q]_{\infty} \quad (1.4)$$

and

$$\chi(-q) = [q; q^2]_{\infty} \quad (1.5)$$