J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.3, No.2 (2014), pp. 79-88

Solutions of the Pell Equation $x^2 - (a^2b^2c^2 + 2ab)y^2 = N$ when $N \in \pm 1, \pm 4$.

V.Sadhasivam, T.Kalaimani and S.Ambika,
PG and Research Department of Mathematics,
Thiruvalluvar Government Arts College,
Rasipuram, Namakkal, Tamil Nadu - 637 401, India.
E.Mail Address: ovsadha@gmail.com, kalaimaths4@gmail.com

Abstract

Let a, b and c be natural numbers and $d = a^2b^2c^2 + 2ab$. In this paper, by using continued fraction expansion of $\sqrt{d}$. We find fundamental solution of the equations $x^2 - (a^2b^2c^2 + 2ab)y^2 = \pm 1$ and we get all positive integer solutions of the equations $x^2 - (a^2b^2c^2 + 2ab)y^2 = \pm 1$ in terms of generalized Fibonacci and Lucas sequences. Moreover, we find all positive integer solutions of the equations $x^2 - (a^2b^2c^2 + 2ab)y^2 = \pm 4$ in terms of generalized Fibonacci and Lucas sequences.

2010 AMS Subject Classification: 11B37, 11B39, 11B50, 11B99, 11A55

Keywords: Diophantine Equations, Pell Equations, Continued Fractions, Generalized Fibonacci and Lucas numbers.

1 Introduction

Let $d \neq 1$ be a positive square free integer and N be any fixed positive integer. Then the equation $x^2 - dy^2 = \pm$ N is known as Pell equation and is named after John Pell(1611-1685), a mathematician who searched for integer solutions to equations of this type in the seventeenth century. For $N = 1$, the Pell equation $x^2 - dy^2 = \pm 1$ is known as classical Pell equation and was studied by Brahmagupta(598-670) and Bhaskara(1114-1185). The Pell equation $x^2 - dy^2 = \pm 1$ has infinitely many solutions (x_n, y_n) for $n \geq 1$. There are several methods for finding the fundamental solutions of Pell's equation $x^2 - dy^2 = 1$ for a positive non square integer "d", e.g. the cyclic method[4] known in India in the 12^{th} century, or the slightly less less efficient but more regular English method (17^{th} century) which produce all solution is based on the simple finite continued fraction expansion of $\sqrt{d}$.

Let $\frac{p_i}{q_i}$ be the sequence of convergence to the continued fraction for $\sqrt{d}$. Then the pair (x_1, y_1) solving Pell's equation and minimizing x satisfies $x_1 = p_i$ and $y_1 = q_i$