J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.3, No.1 (2014), pp. 31-36

Initial-Boundary Value Problems of Fokas' Transform Method

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Received June 5, 2014

Abstract: A new, unified transform method for boundary value problems on linear and integrable nonlinear partial differential equations was recently introduced by Fokas. We consider initial boundary value problems for linear, constant-coefficient evolution equations of arbitrary order on a finite domain. We use Fokas' method to fully characterize well-posed problems. We derive an implicit solution to the IBVP. We also discuss the global relation and classification of boundary condition.
Key words: Unified Transform Method, Nonlinear Partial Differential Equations, IBVP.

1. Introduction

In this paper we develop the major steps of Fokas' unified transform method for solving initial-boundary value problems on linear evolution partial differential equations. In section 2 we define IBVP. Implict solution is given in section 3. And in section 4 we discuss about the global relation and classification of boundary condition. In this paper we assume throughout that the initial-boundary value problem being studied is well-posed in the sense that it admits a unique, smooth solution. Fokas' method does yield a way of checking well-posedness of a problem.

2. The IBVP:

We consider the partial differential equation

$$\partial_t q(x,t) + a(-\iota\partial_x^n)q(x,t) = 0 \quad for (x,t) \in \Omega = [0,1] \times [0,T], \tag{1}$$

where $n \geq 2$ and $a = e^{\iota\theta}$ for some $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ if n is even or $a = \pm\iota$ if n is odd. This choice of a is motivated by our interest in well-posed problems; the reverse heat equation exhibits instantaneous blow-up, for example. We study the initial-boundary value problem on this partial differential equation with initial condition

$$q(x,0) = q_0(x), \tag{2}$$