

On hypergeometric relations among cubic theta functions

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Abstract: In this paper, an attempt has been made to evaluate certain generalized cubic theta functions and also several identities establish by using cubic theta functions.

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1. Introduction, Notations and Definitions

Ramanujan recorded hundreds of modular equations in his three notebooks. Chapters 19 - 21 in Ramanujans second notebook are almost exclusively devoted to modular equations. Ramanujan used modular equations to evaluate class invariants, certain q continued fractions, theta functions and certain other quotients and products of theta functions. Throughout this paper we shall adopt the following notations and definitions

For any number a and q , real or complex and $|q| < 1$,

$$[a; q]_n = [\alpha]_n = \begin{cases} (1 - \alpha)(1 - \alpha q)(1 - \alpha q^2) \dots (1 - \alpha q^{n-1}); & n > 0 \\ 1; & n = 0 \end{cases}$$

Accordingly, we have

$$[a; q]_\infty = \prod_{r=0}^{\infty} (1 - \alpha q^r)$$

Also

$$[a_1, a_2, \dots, a_r; q]_n = [a_1; q]_n [a_2; q]_n \dots [a_r; q]_n.$$

and

$$[a; q]_{-n} = \frac{q^{n(n+1)/2}}{(-a)^n [q/a; q]_n}$$

Motivated with the Jacobi's identity,

$$\theta_3^4(q) = \theta_2^4(q) + \theta_4^4(q) \tag{1.1}$$