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On Certain Special Series and Continued Fractions

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Abstract: In this paper, making use of a known summation formulae for bilateral series, an attempt has been made to establish certain interesting results involving continued fractions.
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1. Introduction, Notations and Definitions

Here and in the sequel, we employ the customary notations for $|q| < 1$,

$$[a;q]_0 = 1, \quad and \quad n \geq 1, \quad let$$

$$[a;q]_n = (1-a)(1-aq)...(1-aq^{n-1}),$$

$$[a;q]_\infty = \prod_{r=0}^{\infty}(1-aq^r)$$

and

$$[a_1, a_2, ..., a_r; q]_n = [a_1;q]_n[a_2;q]_n...[a_r;q]_n.$$

The 'Lost' notebook of Ramanujan contains several results involving Lambert series and continued fraction, we find the following summation formula,

$$\sum_{n=-\infty}^{\infty}\left[\frac{aq^n}{(1-aq^n)^2} - \frac{bq^n}{(1-bq^n)^2}\right] = a\frac{[ab, q/ab, b/a, aq/b; q]_\infty [q;q]_\infty^4}{[a, b, q/a, q/b; q]_\infty^2} \tag{1.1}$$

[Agarwal 1; (4.5) p.197]

It can be utilized to establish several interesting results involving continued fractions. Following results are also needed in our analysis.

$$G(q) = \sum_{n=0}^{\infty}\frac{q^{n^2}}{[q;q]_n} = \frac{1}{[q, q^4; q^5]_\infty}. \tag{1.2}$$