

On Modular Identities and Evaluation of Theta-Functions

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Abstract: . In this paper, making use of modular equations due to Ramanujan relations between α, β and the multiplier m have degree 3,5,7,9,13 and 25 we have established interesting P, Q identities.

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1. Introduction, Notations and Definitions

For real and complex $q, |q| < 1$, then

$$[\alpha; q]_{\infty} = \prod_{k=0}^{\infty} (1 - \alpha q^k),$$

where α is any complex number.

Also,

$$[a_1, a_2, a_3, \dots, a_r; q]_{\infty} = [a_1; q]_{\infty} [a_2; q]_{\infty} \dots [a_r; q]_{\infty}.$$

Ramanujan's defined the general theta function as,

$$f(a, b) = \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2}, \tag{1.1}$$

which by an appeal of Jacobi's triple product identity [Gasper and Rahman 2; App.11 (11.28)] yields,

$$f(a, b) = [ab, -a, -b; q]_{\infty} \tag{1.2}$$

The most important special cases of (1.1) are,

$$\Phi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = \frac{[-q; -q]_{\infty}}{[q; -q]_{\infty}} = [q^2; q^2]_{\infty} [-q; q^2]_{\infty}. \tag{1.3}$$