

On Certain Transformations Involving Theta Hypergeometric Functions

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Abstract: In this paper we attempt to establish certain interesting transformations involving theta hypergeometric functions. We make use of Bailey's lemma and summation of certain truncated theta hypergeometric functions to get our results. These results are the most general ones, leading to a class of transformations of basic hypergeometric functions.

Key words and phrases: Elliptic hypergeometric series, theta hypergeometric functions, transformations, very well-poised theta hypergeometric functions.

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1. Introduction, Notations and Definitions

In a path-breaking paper, Frankel and Turaev [1] introduced elliptic analogues of very well-poised basic hypergeometric series. Elliptic hypergeometric series and their extensions to theta hypergeometric series has become an increasingly active area of research now these days. So far, many formulae for very well-poised basic hypergeometric series have already been extended to the elliptic setting. Some formulae for multi-basic elliptic hypergeometric series appeared in the work of Warnaar [7]. In this paper, using certain identities we have established transformation formulae for the theta hypergeometric series.

A modified Jacobi's theta function with argument x and nome p is defined by,

$$\theta(x; p) = [x; p]_{\infty} [p/x; p]_{\infty} = [x, p/x; p]_{\infty}$$

Also,

$$\theta(x_1, x_2, \dots, x_r; p) = \theta(x_1; p)\theta(x_2; p)\dots\theta(x_r; p)$$

and

$$[x; p]_{\infty} = \prod_{r=0}^{\infty} (1 - xp^r). \quad (1.1)$$