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Hypergeometric relations among Jacobis theta functions

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Abstract: In this paper, an attempt has been made to evaluate and established certain hypergeometric relations containing Jacobi's theta functions.

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Mathematics subject Classification: 33A30, 33D15, 33D20

1. Introduction, Notations and Definitions

Jacobi in 1829 defined following four functions which are called Jacobi's theta – functions;

$$\theta_1(z,q) = 2\sum_{n=0}^{\infty}(-)^n q^{\left(n+\frac{1}{2}\right)^2}\sin(2n+1)z, \tag{1.1}$$

$$\theta_2(z,q) = 2\sum_{n=0}^{\infty} q^{\left(n+\frac{1}{2}\right)^2}\cos(2n+1)z, \tag{1.2}$$

$$\theta_3(z,q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}\cos 2nz, \tag{1.3}$$

and

$$\theta_4(z,q) = 1 + 2\sum_{n=1}^{\infty}(-)^n q^{n^2}\cos 2nz. \tag{1.4}$$

For the absolute convergence of these functions we need $|q| < 1$. Sometimes we use the additional notation $q = e^{\pi i \tau}$, where $\mathrm{Im}(\tau) > 0$.
It is easy to see that

$$\left.\begin{array}{l} \theta_1\left(z+\frac{\pi}{2},q\right) = \theta_2(z), \\ \theta_2\left(z+\frac{\pi}{2},q\right) = -\theta_1(z), \\ \theta_3\left(z+\frac{\pi}{2},q\right) = \theta_4(z) \\ \text{and} \\ \theta_4\left(z+\frac{\pi}{2},q\right) = \theta_3(z). \end{array}\right\} \tag{1.5}$$