J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.2, No.2 (2014), pp. 01-08

On WP-Bailey pairs and Transformation Formulae for Ordinary Hypergeometric Series

S.N. Singh, Rama Jain* and Priyanka Singh,
Department of mathematics,
T.D.P.G. College, Jaunpur-222002 (U.P.) India
*Department of Mathematics,
M.V.P.G. College, Lucknow (U.P.), India

Abstract: In this paper, definition of WP-Bailey pair for ordinary hypergeometric series has been given. Using this definition and some known summations, certain WP-Bailey pairs for ordinary hypergeometric series have been established. These WP-Bailey pairs have been utilized for establishing transformation formulae for ordinary hypergeometric series.
Keywords and Phrases: Ordinary hypergeometric series, WP-Bailey pair, transformation formula and summation formula.
Mathematics subject Classification: 33D15

1. Introduction, Notations and Definitions

The generalized hypergeometric series with r numerator parameters $a_1, a_2, a_3, ..., a_r$ and denominator parameters $b_1, b_2, b_3, ..., b_s$ is defined by

$$ {}_rF_s\left[\begin{matrix} a_1, a_2, ..., a_r; z \\ b_1, b_2, ..., b_s \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n ... (a_r)_n z^n}{(b_1)_n (b_2)_n ... (b_s)_n n!}, \tag{1.1}$$

where

$$(a)_0 = 1, \qquad (a)_n = a(a+1)(a+2)...(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}.$$

The series (1.1), for $r \leq s$, is convergent in the whole complex plane while, for $r = s+1$, it is convergent inside the unit circle $|z| < 1$. The WP-Bailey pair is defined by

$$\beta_n(a,k) = \sum_{r=0}^{n} \frac{(k-a)_{n-r}(k)_{n+r}}{(1)_{n-r}(1+a)_{n+r}} \alpha_r(a,k)$$

$$= \frac{(k-a)_n (k)_n}{n!(1+a)_n} \sum_{r=0}^{n} \frac{(-n)_r (k+n)_r}{(1+a-k-n)_r (1+a+n)_r} \alpha_r(a,k) \tag{1.2}$$