

Basic Analogues of Certain Multiple Series of Transformations-II

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Abstract: In this paper basic analogues of generating relations for certain multiple series with essentially arbitrary terms due to M.A. Pathan, B.B. Jaimini and Shiksha Gautam [1] and Srivastava and Pathan [9] are obtained. The importance of these results lies in obtaining new multiple series transformations and reduction formula which may be capable of yielding number theoretic and combinatorial interpretations.

1. Introduction

Generalizing Heine’s series, we shall define an ${}_r\phi_s$ basic hypergeometric series by

$$\begin{aligned}
 {}_r\phi_s(a_1, a_2, \dots, a_r; b_1, b_2, \dots, b_s; q, z) &\equiv {}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q, z \\ b_1, b_2, \dots, b_s \end{matrix} \right] \\
 &= \sum_{n=0}^{\infty} \frac{(a_1; q)_n (a_2; q)_n \dots (a_r; q)_n}{(q; q)_n (b_1; q)_n \dots (b_s; q)_n} [(-1)^n q^{n(n-1)/2}]^{1+s-r} z^n \tag{1.1}
 \end{aligned}$$

$$(\lambda)_n \equiv (\lambda; q)_n = \left\{ \begin{matrix} 1, \text{ if } n = 0 \\ (1 - \lambda)(1 - \lambda q)(1 - \lambda q^2) \dots (1 - \lambda q^{n-1}) \end{matrix} \right\} \tag{1.2}$$

$$(\lambda; q)_{n+m} = (\lambda; q)_n (\lambda q^n; q)_m \tag{1.3}$$

$$(\lambda; q)_{-n} = \frac{(-\lambda)^{-n} q^{n(n+1)/2}}{(q/\lambda; q)_n} \tag{1.4}$$

$$(\lambda; q^2)_n = (\sqrt{\lambda}; q)_n (-\sqrt{\lambda}; q)_n \tag{1.5}$$

$$(\lambda; q)_{2n} = (\lambda; q^2)_n (\lambda q; q^2)_n \tag{1.6}$$

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} A(k, n) = \sum_{n=0}^{\infty} \sum_{k=0}^n A(k, n - k) \tag{1.7}$$

$$\sum_{n=0}^{\infty} \sum_{k_1 \dots k_r=0}^{M \leq n} \varphi(k_1, k_2 \dots k_r; n) = \sum_{n=0}^{\infty} \sum_{k_1 \dots k_r=0}^{\infty} \phi(k_1 \dots k_r; n + M) \tag{1.8}$$