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On Ramanujan's Theta Functions

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Abstract: In this paper we shall attempt to establish certain Lambert series representations for Ramanujan's theta functions and evaluate certain integrals involving these theta functions with the help of Lambert series representations.
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1. Introduction, Notations and Definitions

Ramanujan's general theta function $f(a,b)$ is defined by

$$f(a,b)=\sum_{-\infty}^{\infty}a^{n(n+1)/2}b^{n(n-1)/2}, \qquad |ab|<1 \tag{1.1}$$

Most important special cases of (1.1) are given by

$$\Phi(q)=f(q,q)=\sum_{n=-\infty}^{\infty}q^{n^2}=\frac{[-q;-q]_\infty}{[q;-q]_\infty}=\frac{[q^2;q^2]_\infty[-q;q^2]_\infty}{[q;q^2]_\infty[-q^2;q^2]_\infty}$$

$$=[q^2;q^2]_\infty[-q;q^2]_\infty, \tag{1.2}$$

$$\Psi(q)=f(q,q^3)=\sum_{n=0}^{\infty}q^{n(n+1)/2}=\frac{[q^2;q^2]_\infty}{[-q;q^2]_\infty} \tag{1.3}$$

$$f(-q)=f(-q,-q^2)=\sum_{n=-\infty}^{\infty}(-)^n q^{n(3n+1)/2}=[q;q]_\infty. \tag{1.4}$$

we shall also have the occasion to use the function $\chi(q)$ given by

$$\chi(-q)=[q;q^2]_\infty, \tag{1.5}$$