

Transformation formulae for poly-basic hypergeometric series

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Abstract: In this paper, we have established some very interesting transformation formulae for poly-basic hypergeometric series.

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1. Introductions, Notations and Definitions:

As usual, for a and q complex numbers with $|q| < 1$, define

$$\begin{aligned}
 [a; q]_0 &= 1 \\
 [a; q]_n &= (1 - a)(1 - aq)(1 - aq^2) \dots (1 - aq^{n-1}), \quad \text{for } n \in N, \\
 [a_1, a_2, a_3, \dots, a_r; q]_n &= [a_1; q]_n [a_2; q]_n [a_3; q]_n \dots [a_r; q]_n, \\
 [a; q]_\infty &= \prod_{r=0}^{\infty} (1 - aq^r).
 \end{aligned}$$

An ${}_r\Phi_s$ basic hypergeometric series is defined by

$$\begin{aligned}
 &{}_r\Phi_s \left[\begin{matrix} a_1, a_2, a_3, \dots, a_r; q; z \\ b_1, b_2, b_3, \dots, b_s \end{matrix} \right] \\
 &= \sum_{n=0}^{\infty} \frac{[a_1, a_2, a_3, \dots, a_r; q]_n}{[q, b_1, b_2, b_3, \dots, b_s; q]_n} [(-)^n q^{n(n-1)/2}]^{1+s-r} z^n. \tag{1.1}
 \end{aligned}$$

A poly-basic hypergeometric series is defined as,

$$\phi \left[\begin{matrix} a_1, a_2, \dots, a_r : c_{1,1}, \dots, c_{1,r_1}; \dots; c_{m,1}, \dots, c_{m,r_m}; q, q_1, \dots, q_m; z \\ b_1, b_2, \dots, b_s : d_{1,1}, \dots, d_{1,s_1}; \dots; d_{m,1}, \dots, d_{m,s_m} \end{matrix} \right]$$