

Reducibility of ordinary double hypergeometric functions

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Abstract: In this paper, ordinary double hypergeometric series have been reduced into single series.

Keywords and Phrases: Ordinary hypergeometric series, summation/product formula/ double ordinary hypergeometric series.

1. Introduction, Notation and Definition In this paper, we have established certain results involving double hypergeometric series and single hypergeometric series. Certain product formulae have been deduced from the results. Results established in this paper are quite interesting and useful. An explicit representation of the generalized hypergeometric series is given by

$${}_rF_s \left[\begin{matrix} a_1, a_2, a_3, \dots, a_r; z \\ b_1, b_2, b_3, \dots, b_s \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_r)_n z^n}{(b_1)_n (b_2)_n \dots (b_s)_n (1)_n}, \quad (1.1)$$

valid for $|z| < 1$, provided no zeros appear in the denominator. Here $a_1, a_2, a_3, \dots, a_r$ and $b_1, b_2, b_3, \dots, b_s$ and z are assumed to be complex numbers.

The shifted factorial is denoted by

$$(a_n) = \begin{cases} 1, & n = 0 \\ a(a+1)\dots(a+n-1), & n \geq 1 \end{cases} \quad (1.2)$$

The generalized ordinary double hypergeometric series is given as,

$$\begin{aligned} F_{l:m;s}^{p:q;t} \left[\begin{matrix} (a_p) : (b_q); (c_t); x, y \\ (\alpha_l) : (\beta_m); (\gamma_s) \end{matrix} \right] &= F \left[\begin{matrix} (a_p) : (b_q); (c_t); x, y \\ (\alpha_l) : (\beta_m); (\gamma_s) \end{matrix} \right] \\ &= \sum_{n,r=0}^{\infty} \frac{[(a_p)]_{n+r} [(b_q)]_n [(c_t)]_r x^n y^r}{[\alpha_l]_{n+r} [\beta_m]_s [\gamma_s]_r n! r!} \end{aligned} \quad (1.3)$$