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On Certain Identities involving q-Series

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Abstract: In this paper, making use of a transformation formula of ${}_2\Psi_2$ due to Bailey [2]. We have established certain Eta function identities, which compliment the works of Berndt and Zhang [3] Bhargava and Somashekara [4] Fine [5].
Keywords: Bilateral basic hypergeometric series, Identities, Eta function identities.
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1. Introduction, Notations and Definitions:

In this paper we establish certain Eta function identities which compliment the works of Berndt and Zhang [3] Bhargava and Somashekara [4] Fine [5].
We shall use the following known transformation due to Bailey [2]

$$ {}_2\Psi_2\left[\begin{array}{l} a, b; q; z \\ c, d \end{array}\right] = \frac{[az, d/a, c/b, dq/abz; q]_\infty}{[z, d, q/b, cd/abz; q]_\infty}\ {}_2\Psi_2\left[\begin{array}{l} a, abz/d; q; d/a \\ az, c \end{array}\right] \tag{1.1}$$

to establish our results involving Eta function.
Taking $z = q/a$ and $b = d/q$ in (6.1.1) we get after some simplification.

$$\sum_{n=-\infty}^{\infty} \frac{[a, d/q; q]_n (q/a)^n}{[c, d; q]_n} = \frac{[q, d/a, cq/d, q; q]_\infty}{[q/a, d, q^2/d, c; q]_\infty} \tag{1.2}$$

Now replacing d by dq in (1.2), we get

$$\sum_{n=-\infty}^{\infty} \frac{[a; q]_n (q/a)^n}{[c; q]_n (1 - dq^n)} = \frac{[q, dq/a, c/d, q; q]_\infty}{[q/a, d, q/d, c; q]_\infty} \tag{1.3}$$

As $a \to \infty$ in (1.3), we get

$$\sum_{n=-\infty}^{\infty} \frac{(-)^n q^{n(n+1)/2} (1 - dq^n)}{[c; q]_n} = \frac{[c/d; q]_\infty [q; q]_\infty^2}{[c, d, q/d; q]_\infty} \tag{1.4}$$