

On certain results involving the product of partial theta functions

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(Received October 09, 2012)

Abstract: In this paper, we have established a new identity for the product of two partial theta functions. Certain interesting results have been deduced as the special cases of the main identity.

Keywords and Phrases: Partial theta functions, Jacobi’s triple product identity, Bailey’s transform, false theta function.

2000 Mathematics subject classification: 33D15

1. Introduction

We will use the standard q- notations,

$$[a; q]_{\infty} = \prod_{n=1}^{\infty} (1 - aq^{n-1}),$$

$$[a_1, a_2, \dots, a_r; q]_{\infty} = [a_1; q]_{\infty} [a_2; q]_{\infty} \dots [a_r; q]_{\infty},$$

$$[a; q]_0 = 1,$$

and

$$[a; q]_n = (1 - a)(1 - aq)(1 - aq^2) \dots (1 - aq^{n-1}) \text{ for } n \geq 1.$$

Warnaar [6] proved the following generalization of Jacobi’s triple product identity,

$$1 + \sum_{n=1}^{\infty} (-)^n q^{n(n-1)/2} (a^n + b^n) = [q, a, b; q]_{\infty} \sum_{n=0}^{\infty} \frac{\left[\frac{aq}{q}; q \right]_{2n} q^n}{[q, a, b, ab; q]_n}. \quad (1.1)$$

The celebrated Jacobi triple product identity [3; Eq. (1.6.1) page 15] is

$$\sum_{n=-\infty}^{\infty} (-)^n a^n q^{n(n-1)/2} = [q, a, q/a; q]_{\infty}. \quad (1.2)$$