

On Certain new identities associated with Ramanujan's modular equations

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Abstract: In this paper, making use of certain modular equations due to Ramanujan, an attempt has been made to establish some new P-Q identities.

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1. Introduction, Notations and Definitions:

Ramanujan has recorded several P-Q eta function identities in chapter 25 of his second notebook. Berndt [4] and Berndt and Zhang [6] proved these P-Q identities by employing various modular equations belonging to the classical theory. In this paper, an attempt has been made to establish certain P-Q identities by making use of modular equations due to Ramanujan. Many other mathematicians have used P-Q eta function identities to evaluate Ramanujan's theta functions, Ramanujan's class invariants, Rogers-Ramanujan continued fraction. In this regard the works of Berndt and Chan [7], Berndt, Chan and Zhang [9], Bhargava and Adiga [10] are note worthy.

Let $Z_r = Z(r, x) = {}_2F_1[1/r, (r-1)/r; 1; x]$ and

$$q_r = q_r(x) = \exp\left(-\pi \operatorname{cosec}(\pi/r) \frac{{}_2F_1[1/r, (r-1)/r; 1; 1-x]}{{}_2F_1[1/r, (r-1)/r; 1; x]}\right), \quad (1.1)$$

where $r = 2, 3, 4$ and 6 and ${}_2F_1[1/r, (r-1)/r; 1; x]$ denotes the ordinary hypergeometric functions defined as;

$${}_2F_1[a, b; c; z] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n z^n}{(c)_n n!},$$