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Some Characteristics of Hip Joint

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Abstract: We have presented expressions of pressure and load capacity for certain values of squeezing velocity, concentration index of micromolecules, slip parameter and design parameter regarding some characteristics of hip joints. The coefficient of friction is obtained as $f = F/W$. These results are given in figures (1.2) and (1.3) with respect to coefficient of pressure α. It is demonstrated that the pressure increases with respect to α variation. The pressure further increases for each values of α due to the squeezing action. It is expected that pressure if the concentration of micromolecule increases. It is obvious from figure (1.3), load capacity increases as α increases with decrease in slip parameter. The increase in load capacity and pressure with α and decrease in slip parameter implies the increase in the coefficient of friction, which is a good result.
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AMS subject classification code:

1. Introduction:

Mak and Conway (1977), Kapur and Yadav (1981) studied characteristics of hip joint for engineering problems. We have presented some characteristics of hip joint depending on the choice of lubrication mechanism, rheological features and also the material features of the cartilage. The interaction of rheological feature of fluid and material features such as elasticity and permeabilities was investigated by Higginson (1974) and elastohydrodynamic lubrication method was presented by Dintenfoss (1970). We have presented the problem of hip joint under the similar conditions by neglecting the elasticity effect of the cartilage at first state to understand the pressure generation. Hence, in this chapter we have analysised the hip joint.

2. Equations and Boundary Conditions:

The joint geometry is given in figure (1.1) with following equations of motion

$$(K + \mu_\circ e^{\alpha' P})\frac{\partial^2 u'}{\partial z'^2} + K\frac{\partial v'}{\partial z'} = \frac{dp}{dr'}, \tag{2.1}$$

$$\gamma_v \frac{\partial^2 v'}{\partial z'^2} - K\left(\frac{\partial u'}{\partial z'} + 2v'\right) = 0, \tag{2.2}$$

and continuity equation reads

$$\frac{\partial(u'r')}{\partial r'} + \frac{\partial(v'r')}{\partial z'} = 0. \tag{2.3}$$

The boundary conditions in region I

$$\bar{P} = 0 \qquad \text{at} \quad r' = r_\circ, \tag{2.4}$$

$$\frac{\partial \bar{P}}{\partial z'} = 0 \qquad \text{at} \quad z' = -H', \tag{2.5}$$

where H‘ be the thickness of porous pad.
The boundary conditions in region II i.e. non porous region read

$$u' = -\frac{\sqrt{k'}}{\Delta}\frac{\partial u'}{\partial z'}, \tag{2.6}$$

$$v' = -\frac{k'}{\mu'}\frac{\partial \bar{p}}{\partial z'} \qquad \text{at} \quad z' = 0, \tag{2.7}$$

$$v' = 0 \tag{2.8}$$

$$u' = 0 \tag{2.9}$$

$$v' = -\frac{dh'}{dt} \quad \text{at} \quad z' = h' \tag{2.10}$$

$$v' = 0 \tag{2.11}$$

The region III i.e. interface condition

$$\bar{p} = p \quad \text{at} \quad z' = 0, \tag{2.12}$$

$$\bar{v}' = v' = -\frac{k'}{\mu'}\frac{\partial \bar{p}}{\partial z'} \quad \text{at} \quad z' = 0 \tag{2.13}$$

Let us define non-dimensional quantities

$$r = \frac{r'}{r_\circ}, \tag{2.14}$$

$$z = \frac{z'}{h_\circ}, \tag{2.15}$$

$$u = \frac{u'}{U_\circ}, \tag{2.16}$$

$$v = \frac{v' h_\circ}{U_\circ}, \tag{2.17}$$

$$\lambda = \frac{K}{\mu_\circ}, \tag{2.18}$$

$$P = \frac{p h_\circ^2}{\mu_\circ U_\circ r_\circ}, \tag{2.19}$$

$$\sigma = \frac{\sqrt{k'}}{\Delta h_\circ}, \tag{2.20}$$

$$H = \frac{H'}{h_\circ}, \tag{2.21}$$

$$\gamma = \frac{\gamma_\circ}{K h_\circ^2}, \tag{2.22}$$

$$\psi = -\frac{k' H'}{h_\circ^3}, \tag{2.23}$$

$$\alpha = \alpha' \frac{\mu_\circ U_\circ r_\circ}{h_\circ^2}. \tag{2.24}$$

Hence, the equations of motion in non-dimensional from reads

$$(e^{\alpha' P}+\lambda)\frac{\partial^2 u}{\partial z^2}-\lambda\frac{\partial v}{\partial z}=\frac{dP}{dr}, \tag{2.25}$$

$$\gamma\frac{\partial^2 v}{\partial z^2}-\lambda\frac{\partial u}{\partial z}=2v, \tag{2.26}$$

$$\frac{\partial(ur)}{\partial r}+\frac{\partial(vr)}{\partial z}=0, \tag{2.27}$$

where

$$\mu=\mu_\circ e^{\alpha P}. \tag{2.28}$$

The boundary condition read as;
In region I

$$\bar{P}=0 \qquad \text{at} \qquad r=\pm 1, \tag{2.29}$$

$$\frac{\partial \bar{P}}{\partial z}=0 \qquad \text{at} \qquad z=-H. \tag{2.30}$$

In region II

$$u=-\sigma\frac{\partial u}{\partial z} \tag{2.31}$$

$$\bar{v}=v=-\frac{\psi}{H}e^{-\alpha P}\frac{\partial \bar{P}}{\partial z}, \tag{2.32}$$

$$v=0 \qquad \text{at} \qquad z=0, \tag{2.33}$$

$$u=0 \tag{2.34}$$

$$v=-\frac{dh}{dt}, \tag{2.35}$$

$$v=0 \qquad \text{at} \qquad z=h. \tag{2.36}$$

In region III

$$\bar{P}=P \qquad \text{at} \qquad z=0, \tag{2.37}$$

$$\bar{v}=v=-\frac{\psi}{\mu}e^{-\alpha P}\frac{\partial P}{\partial z} \qquad \text{at} \qquad z=0. \tag{2.38}$$

3. Solutions:

Now one may obtain radial velocity u and the spin velocity v with the help of boundary conditions.

$$u=-\frac{1}{(\lambda+2e^{\alpha P})S}\frac{dP}{dr}[2hT(m\sigma T+2\sigma-hr)cosh\ mz$$

$$-2hT^2 sinh\ mz + 2hT^2 sinh(z-h)m - 2h^2 T cosh(z-h)m$$
$$+2hT(2z-h-2\sigma m\sigma T)cosh\ mh + 2h(T^2+2hz-2h^2)sinh\ mh$$
$$+6h^2T - 4zhT - (z^2-h)S], \tag{3.1}$$

$$v = \frac{1}{(\lambda+2e^{\alpha P})S}\frac{dP}{dr}[(2m\sigma Th + 4\sigma h - 2h^2)sinh\ mz$$
$$-2hTcosh\ mz + 2hTcosh(z-h)m$$
$$-2h^2 sinh(z-h)m - 2hTcosh\ mh - 2h^2 sinh\ mh + Sz], \tag{3.2}$$

where

$$S = 4(cosh\ mh - 1)T + (4h - 4\sigma - 2m\sigma T)sinh\ mh, \tag{3.3}$$

$$m = \frac{\sqrt{\lambda+2e^{\alpha P}}}{\gamma(\lambda+2e^{\alpha P})}, \tag{3.4}$$

$$T = \gamma m - \frac{2}{m}. \tag{3.5}$$

Now, we obtain

$$\bar{v} = -\frac{\psi}{H}\frac{\partial \bar{P}}{\partial z}\ |_{z=0}\ e^{-\alpha P} \tag{3.6}$$

which assumes the form

$$\bar{v} = -\psi e^{-\alpha P}\left(-\frac{\partial^2 P}{\partial r^2} - \frac{1}{r}\frac{\partial \bar{P}}{\partial r} + \alpha\left(\frac{\partial P}{\partial r}\right)^2\right), \tag{3.7}$$

$$v\ |_{z=0} = \bar{v} = \psi e^{-\alpha P}\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{P}}{\partial r} - \alpha\left(\frac{\partial P}{\partial r}\right)^2\right], \tag{3.8}$$

After doing some mathematics, the pressure distribution reads

$$P = -\frac{C_2}{2C_1} \pm \frac{\sqrt{C_2^2 - 4C_1C_3}}{2C_1} \tag{3.9}$$

where

$$C_1(h,\psi,\sigma,\alpha,\lambda) = -\left[\alpha\psi(\sigma-h) - \frac{1}{4}\frac{h^3\sigma\alpha}{(\lambda+1)^2} - \frac{\alpha h^3(\sigma-h)}{(\lambda+2)^2}\right], \tag{3.10}$$

$$C_1(h,\psi,\sigma,\lambda) = \left[2\psi(\sigma-h) - \frac{h^3}{6\lambda}\left\{(5\sigma-2h) - \frac{3(6\sigma-4h+5\lambda\sigma-4h\lambda)}{(\lambda+1)(\lambda+2)}\right\}\right], \tag{3.11}$$

$$C_3(h,\sigma,r) = (\sigma - h)(r^2 - 1)\frac{dh}{dt}. \tag{3.12}$$

Again the load capacity W as

$$W = \int_0^1 2\pi P r dr \tag{3.13}$$

or,

$$W = \pi - \left[\frac{C_2}{2C_1}\right] - \frac{2\pi C_1}{3(h-\sigma)\frac{dh}{dt}} \times \left[\left(\frac{C_2}{2C_1}\right)^3 - \left(\frac{C_2}{4C_1^2} + \frac{(\sigma - h)}{C_1}\frac{dh}{dt}\right)^{3/2}\right]. \tag{3.14}$$

4. Concluding Remarks: We have presented expressions of pressure and load capacity for certain values of squeezing velocity, concentration index of micro-molecules, slip parameter and design parameter regarding some characteristics of hip joints. The coefficient of friction is obtained as $f = F/W$. These results are given in figures (1.2) and (1.3) with respect to coefficient of pressure α. It is demonstrated that the pressure increases with respect to α variation. The pressure further increases for each values of α due to the squeezing action. It is expected that pressure if the concentration of micromolecule increases. It is obvious from figure (1.3), load capacity increases as α increases with decrease in slip parameter. The increase in load capacity and pressure with α and decrease in slip parameter implies the increase in the coefficient of friction, which is a good result.

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