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Lie Theoretic Study of some generating Functions of $F_{mn+\nu}^{m}(x;\lambda,\nu)$

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ABSTRACT :
In this paper generating functions for the orthogonal polynomials $F_{mn+\nu}^{m}(x;\lambda,\nu)$ are obtained by considering the irreducible representation $\uparrow_{w,\mu}$ of $\mathcal{G}(1,0)$ by Miller's method .

Keywords : Generating Functions

A. M. Subject Classification : 33C45 , 33 C 80

1. INTRODUCTION :

Dutta and Manocha [2] developed an interesting class of orthogomal polynomials

$$F_{mn+\nu}^{m}(x;\lambda,\nu) = x^{\nu}\,{}_1F_1[-n;(2\nu+\lambda+m-1)/m\,;\,x^m] \qquad (1.1)$$

which satisfies the differential equation

$$x^2\frac{d^2y}{dx^2} - (mx^{m+1} - \lambda x)\frac{dy}{dx} + y\{m(mn+\nu)x^m - \nu(\nu+\lambda-1)\} = 0 \qquad (1.2)$$

where λ , m are fixed parameters , n is a variable parameter and ν is a non - negetive integer < m . The polynomials $F_{mn+\nu}^{m}(x;\lambda,\nu)$ satisfy the following differential recurrence relations

$$xDF_{mn+\nu}^{m}(x;\lambda,\nu) = (\nu+mn)F_{mn+\nu}^{m}(x;\lambda,\nu) - mnF_{m(n-1)+\nu}^{m}(x;\lambda,\nu) \qquad (1.3)$$

and

$$xDF_{mn+\nu}^{m}(x;\lambda,\nu) = (2\nu+m+mn+\lambda-1)F_{m(n+1)+\nu}^{m}(x;\lambda,\nu) - (mn+m+\nu+\lambda-1-mx^m)F_{mn+\nu}^{m}(x;\lambda,\nu) \qquad (1.4)$$

The object of the present paper is to study $F_{mn+\nu}^{m}(x;\lambda,\nu)$ by giving a suitable interpretation to n and then to derive new classes of generating functions with the help of Lie theoretic method of Miller [4] .

2. GROUP THEORETIC DISCUSSION :

For any pair of complex numbers (a , b) we define the 4th dimensional complex Lie- algebra $\mathcal{G}(a,b)$ with basis g^+ , g^- , g^3, ε by

$$[g^+, g^-] = 2a^2g^3 - b\varepsilon,\ [g^3, g^+] = g^+,\ [g^3, g^-] = -g^-, \qquad (2.1)$$
$$[g^+, \varepsilon] = [g^-, \varepsilon] = [g^3, \varepsilon] = \theta$$

where [,] is the commutator bracket and θ is the additive identity element .