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On evaluation of certain Ramanujan's theta functions

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and
$$\chi(-q) = [q; q^2]_\infty. \tag{6}$$

Abstract: In this paper, we have established modular identities with the help of Ramanujan's modular equations and using these identities we have evaluated certain theta functions defined by Ramanujan.

Keywords and Phrases: modular equation/ modular identity/ theta function.

AMS subject classification code: 33A15.

Introduction, Notations and Definitions

In the Second Notebook of Ramanujan [3], he has recorded several results in the theories of elliptic functions to alternative bases. In Entries 51 - 72 of chapter 25 of the Second Notebook, Ramanujan states twenty three elegant P - Q identities. The main purpose of this paper is to establish certain P - Q identities by making use of modular equations found in the Second Notebook and then to evaluate Ramanujan's theta functions for some particular values of q by using these P - Q identities.

For real or complex q ($|q| < 1$), let $[\alpha; q]_\infty = \prod_{k=0}^{\infty}(1 - \alpha q^k)$, where α is any complex number.

Also,
$[a_1, a_2, ..., a_r; q]_\infty = [a_1; q]_\infty [a_2; q]_\infty ... [a_r; q]_\infty$.

Ramanujan defined the general theta function as,
$$f(a, b) = \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2}, \tag{1}$$
which by an appeal of Jacobi's triple product identity [2; App. II (II.28)],yields
$$f(a, b) = [ab, -a, -b; ab]_\infty. \tag{2}$$
The most important special cases of (1) are,
$$\phi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = \frac{[-q; -q]_\infty}{[q; -q]_\infty} = [q^2; q^2]_\infty [-q; q^2]^2_\infty, \tag{3}$$
$$\psi(q) = \sum_{n=0}^{\infty} q^{n(n+1)/2} = \frac{[q^2; q^2]_\infty}{[q; q^2]_\infty}, \tag{4}$$
$$f(-q) = \sum_{n=-\infty}^{\infty} (-)^n q^{n(3n-1)/2} = [q; q]_\infty, \tag{5}$$