J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.1, No.1 (2012), pp.29-46

On Bailey pair and q-series transformations

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Abstract: In this paper, making use of the definition of Bailey pair and theorems for constructing new Bailey pair from a known pair, we have established transformation formulae for q-series.
Keywords: Bailey pair, transformation formula, identity.
AMS subject classification code: 33D15

1. Notations and Definitions As usual, for a and q complex numbers with $|q| < 1$, define

$$[a;q]_0 = 1,$$

$$[a;q]_n = (1-a)(1-aq)...(1-aq^{n-1})$$

for $n \in N$,

$$[a_1, a_2, ..., a_k; q]_n = [a_1;q]_n[a_2;q]_n...[a_k;q]_n.$$

and

$$[a;q]_\infty = \prod_{r=0}^{\infty}(1-aq^r)$$

A basic hypergeometric series is defined by,

$$ {}_r\Phi_s \left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{array} \right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_n z^n}{[q, b_1, b_2, ..., b_s; q]_n} \left[(-)^n q^{n(n-1)/2} \right]^{1+s-r}. \tag{1.1}$$

we shall make use of following summation formulae in our analysis.

$$ {}_4\Phi_3 \left[\begin{array}{l} a, -q\sqrt{a}, b, q^{-n}; q; \dfrac{a^{1/2}q^{1+n}}{b} \\ -\sqrt{a}, aq/b, aq^{1+n} \end{array} \right] = \frac{\left[aq, qa^{1/2}/b; q\right]_n}{\left[qa^{1/2}, aq/b; q\right]_n}. \tag{1.2}$$

[Gasper and Rahman 3; App II (II. 14)]

$$ {}_6\Phi_5 \left[\begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}, b, c, q^{-n}; q; \dfrac{aq^{1+n}}{bc} \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq^{1+n} \end{array} \right] = \frac{[aq, aq/bc; q]_n}{[aq/c, aq/b; q]_n}. \tag{1.3}$$

[Gasper and Rahman 3; App II (II. 21)]

2. Introduction

A pair of sequences $\{\alpha_n, \beta_n\}$ satisfying the relation

$$\beta_n = \sum_{r=0}^{n} \frac{\alpha_r}{[q;q]_{n-r}[aq;q]_{n+r}} = \frac{1}{[q, aq; q]_n} \sum_{r=0}^{n} \frac{[q^{-n};q]_r(-)^r q^{nr}}{q^{r(r-1)/2}[aq^{n+1};q]_r} \alpha_r. \tag{2.1}$$