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Evaluations of Ramanujan's Cubic Continued Fraction $G(q)$ by Modular Equations

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Abstract

In this paper. we establish certain modular equations related to Ramanujan's cubic continued fraction

$$G(q) := \frac{q^{1/3}}{1} {}_{+} \frac{q+q^2}{1} {}_{+} \frac{q^2+q^4}{1} {}_{+\ldots}, \qquad |q| < 1.$$

and obtain many explicit values of $G(e^{-\pi\sqrt{n}})$, for certain values of n.

Keywords and Phrases: Ramanujan cubic continued fraction, Theta functions, Modular Equations.

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1 Introduction

Ramanujan's cubic continued fraction $G(q)$ is defined by

$$G(q) := \frac{q^{1/3}}{1} {}_{+} \frac{q+q^2}{1} {}_{+} \frac{q^2+q^4}{1} {}_{+\ldots}, \qquad |q| < 1. \tag{1.1}$$

Ramanujan recorded this continued fraction (1.1) on page 227 of his third notebook [10] and on page 44 of his lost notebook [11]. For the wonderful introduction to Ramanujan's continued fraction see [3], [5], [9] and for some beautiful subsequent work on Ramanujan's cubic contnued fraction [1], [2],[4]. [12], [13].

In this paper, we establish certain general formulae for evaluating $G(q)$. In section 2 of this paper, we setup some preliminaries which are required to prove the general formulae. In section 3, we establish certain modular equations related to $G(q)$ and in the final section, we deduce the above stated general formulae and obtain many explicit values of $G(q)$.

We conclude this introduction by recalling an identity for $G(q)$ stated by Ramanujan.

$$8G^3(-q) = 1 - \frac{\varphi^4(q)}{\varphi^4(q^3)}, \tag{1.2}$$