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On Reduction and Product Formulae for q- Series

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Abstract: In this paper we have established certain reduction and product formulae for basic hypergeometric series.
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1. Introduction, Notations and Definitions

As usual, let for any positive integer n,

$$[\alpha; q^k]_n = (1-\alpha)(1-\alpha q^k)...(1-\alpha q^{k(n-1)}),$$

$$[\alpha; q^k]_0 = 1,$$

$$[\alpha; q^k]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^{nk}),$$

and

$$[a_1, a_2, ..., a_r; q^k]_n = [a_1; q^k]_n [a_2; q^k]_n ... [a_r; q^k]_n.$$

we define a generalized basic or q-hypergeometric series as,

$$ {}_r\phi_s \left[\begin{array}{l} a_1, a_2, ..., a_r; q^k; z \\ b_1, b_2, ..., b_s \end{array} \right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q^k]_n z^n}{[q^k, b_1, b_2, ..., b_s; q^k]_n} [(-)^n q^{kn(n-1)/2}]^{(1+s-r)}, \tag{1.1}$$

where $q \neq 0$ and $r \leq s+1$.
The series (1.1) converges absolutely for all z, if $r \leq s$ and for $|z| < 1$ if $r = s+1$.
Following Agrawal and Verma [1], we define the bibasic hypergeometric series as,

$$\phi \left[\begin{array}{l} (a) : (b); x \\ (c) : (d); q^\lambda \end{array} \right] = \sum_{n=0}^{\infty} \frac{[(a); q^2]_n [(b); q]_n x^n q^{\lambda n(n-1)/2}}{[(c); q^2]_n [(d); q]_n}, \tag{1.2}$$

where (a) stands for the sequence of parameters $a_1, a_2, ..., a_A$.
The series (1.2) converges for all values of x if $\lambda > 0$ and when $\lambda = 0$, we require the additional convergence condition $|x| < 1$.
We shall make use of the following identity in our analysis

$$\sum_{n=0}^{\infty}\sum_{r=0}^{n} A(n, r) = \sum_{n,r=0}^{\infty} A(n+r, r). \tag{1.3}$$