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A note on Partition and Continued Fractions

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Amit Kumar Singh and Vijay Yadav, Department of Mathematics,
T.D.P.G. College, Jaunpur-222002 (U.P.) India

Abstract: In this paper we have established the continued fraction representations of different partition generating functions.

Keywords and Phrases: Continued fraction, partition generating function, partition

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1. Introduction

Recently, Singh, S.N. [4] and Denis, R.Y. [1,2] have derived a variety of continued fraction expressions for the quotients of two ${}_2\phi_1$- series. In this paper making use of the following result of Singh, S.N. [3], an attempt has been made to represent the different generating functions of partitions in the form of continued fractions.

$$\left(1-\frac{\gamma}{q}\right)\frac{{}_2\phi_1\,(\alpha,\beta;\gamma;x)}{{}_2\phi_1\,(\alpha,\beta q;\gamma;x)}=\left(1-\frac{\gamma}{q}\right)-\frac{x\beta(1-\alpha)}{\dfrac{(1-\gamma/\beta q)}{(1-\gamma/q)}+}$$

$$\frac{(\gamma/q\beta)(1-\beta q)(1-\alpha\beta xq/\gamma)}{(1-\gamma/q)-}\,\frac{x\beta q(1-\alpha q)}{\dfrac{(1-\gamma/\beta q)}{(1-\gamma/q)}+}\,\frac{(\gamma/q\beta)(1-\beta q^2)(1-\alpha\beta xq^2/\gamma)}{(1-\gamma/q)-}\cdots \tag{1.1}$$

2. Notations and Definitions

The generalized basic hypergeometric function

$${}_A\phi_B\left[\begin{array}{l}(a);q;z\\(b);i\end{array}\right]=\sum_{r=0}^{\infty}q^{ir(r-1)/2}\frac{\prod_{j=1}^{A}(a_j;q)_r z^r}{\prod_{j=1}^{B}(b_j;q)_r(q;q)_r}, \tag{2.1}$$

where $\quad (a;q)_r=(1-a)(1-aq)...(1-aq^{r-1})$

$$(a;q)_0=1$$

$i>0, |q|<1, |z|<\infty$ and for i=0, max. $(|q|,|z|)<1$. Also (a) stands for sequence of A-parameters of the form $a_1,a_2,...,a_A$.
The generating function for the partitions of any positive integer given by Euler is as;

$$\frac{1}{(q;q)_\infty}=1+\sum_{n=1}^{\infty}p(n)q^n, \tag{2.2}$$

where p(n) stands for the number of unrestricted partitions of positive integer n.

Also,
$$\frac{1}{(q;q)_m}=1+\sum_{n=1}^{\infty}p_m(n)q^n, \tag{2.3}$$