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Double integral representation and certain transformations for basic Appell functions

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Abstract:
In the present paper we have studied basic analogue of Appell's hypergeometric functions called q-Appell functions and express the functions Φ^1, Φ^2, Φ^3 in terms of definite integrals. Also certain transformation formulae have been obtained related these functions. Some special cases have been also discussed.
Keywords and Phrases: q-Appel functions, Integral representation and reduction formula.
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1. Introduction

The basic analogue of Appell's hypergeometric functions of two variables were first defined and studied by F.H. jackson [3 and 4], R.P. Agarwal [1 and 5] also studied these functions and gave some general identities involving these functions. G.E. Andrews [6] also studied these functions and showed that the Appell function Φ^1 can be reduced to the series ${}_3\Phi_2$. Sharma and jain [7] studied that q-Appell functions have been brought within the preview of Lie-theory by deriving reduction formulae for q-Appell functions namely Φ^1 and Φ^2 using the dynamical symmetry algebra of basic hypergeometric function ${}_2\Phi_1$.
We shall use the following usual basic hypergeometric notations for $|q| < 1$,
$(a;q)_n = (1-a)(1-aq)(1-aq^2)...(1-aq^{n-1})$
and $(a)_\infty = \lim\limits_{n\to\infty} (a)_n$

$$\lim_{q\to 1}(q;q)_n = (1)_n = n!, \quad \text{and} \quad (a;q)_0 = 1. \tag{1.1}$$

$$\Phi^1[a;b;b';c;x;y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(a)_{m+n}(b)_n}{(q)_m(q)_n(c)_{m+n}} x^m y^n \tag{1.2}$$

$$\Phi^2[a;b;b';c;c'x;y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(a)_{m+n}(b)_m(b')_n}{(q)_m(q)_n(c)_m(c')_n} x^m y^n \tag{1.3}$$

$$\Phi^3[a;a';b;b';c;x;y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(a)_m(a')_n(b)_m(b')_n}{(q)_m(q)_n(c)_{m+n}} x^m y^n \tag{1.4}$$

$$\Phi^4[a;b;c;c'x;y] = \sum_{n=0}^{\infty}\sum_{n=0}^{\infty} \frac{(a)_{m+n}(b)_{m+n}}{(q)_m(q)_n(c)_m(c')_n} x^m y^n \tag{1.5}$$