

ON HYPERGEOMETRIC PROOF OF CERTAIN CONTINUED FRACTION RESULTS

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Abstract :

In this paper, we provide hypergeometric proof of certain results and deduce a number of new and known results. This result is equivalent to Entry 12 of Chapter XVI of Ramanujan's second Notebook.

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1. INTRODUCTION, NOTATION AND DEFINITION

Ramanujan's contribution to continued fractions associated with analytic functions is remarkable. His Notebooks contain a large number of beautiful results associated with hypergeometric functions (both, basic and ordinary) and continued fractions. Many of his continued fraction results can be provided with hypergeometric proof. In a recent publication Denis and Singh [2, 3] provided hypergeometric proof of Entries 25 and 33 of Chapter XII of Ramanujan's [5] Second Notebook, and also provided their basic analogues.

Motivated by the above results, we propose to provide hypergeometric proof of the following results.

$$\frac{[a^2q^3, b^2q^3; q^4]_\infty}{[a^2q, b^2q; q^4]_\infty} = \frac{1}{1 - a^2q} - \frac{q(b^2 - a^2q^2)}{1 + q^2} + \frac{q(a^2 - b^2q^2)}{(1 - a^2q)(1 + q^4)} - \frac{(q^5(b^2 - a^2q^6))}{1 + q^6} + \frac{q(a^2 - b^2q^6)}{+(1 - a^2q)(1 + q^8)} - \frac{q^9(b^2 - a^2q^{10})}{1 + q^{10} + \dots\dots} \tag{1.1}$$

where $[\alpha, \beta; p]_\infty = [\alpha; p]_\infty [\beta; p]_\infty$ and $[\alpha; p]_\infty = \prod_{r=0}^{\infty} (1 - \alpha p^r), |p| < 1.$