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On certain Transformation Formula of Ordinary Hypergeometric Series using laplace transform

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Dr. Jay Prakash Yadav, Dr. N.N. Pandey, S.N. Hatekar, & Sunil Singh,*
Department of Mathematics & statistics, PDL College of Commerce and Economics,
Malad (W), Mumbai.
*Department of Mathematics & Statistics,
Sydenham College of Commerce and Economics,
Mumbai.

Abstract:
This is a brief survey of theory of hypergeometric series with particular emphasis on establishing some transformation formula in hypergeometric series using Laplace transform and Pochamar's integral.

Keywords and Phrases: Hypergeometric series, summation formula and Laplace transform.

Notations and Known results

An infinite series of the form $\sum_{n=0}^{\infty} \frac{(a_n)(b_n)}{(c_n)(1_n)} z^n$ is known as ordinary hypergeometric series. It is denoted as ${}_2F_1[a, b; c; z]$. So,

$$ {}_2F_1[a, b; c; z] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} z^n \tag{1}$$

Where a,b and c are real or complex parameters and z is an argument with $|z| < 1$ and

$$(a_n) = a(a + 1)(a + 2)...(a + n - 1).$$

The Pochamer integral is given by

$$ {}_2F_1[a, b; c; z] = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a} dt \tag{2}$$

For $Re(b) > 0$ and $Re(c - a - b) > 0$.

The integral (2) gives

$$ {}_2F_1[a, b; c; 1] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \tag{3}$$

This is known as gauss Summation formula. The restriction $Re(b) > 0$ is not necessary for this formula to hold. Gauss [1813] used one of his contiguous relation to prove (3) is hold if $Re(c - a - b) > 0$.

$$ {}_2F_1[-n, b; c; 1] = \frac{(c-b)_n}{(c)_n}. \tag{4}$$

This is simply an alternative form of the discrete binomial formula.

$$(a + b)_n = \sum_{k=0}^{n} (n, k)(a)_k (b)_{n-k} \tag{5}$$